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A tabu search algorithm for solving economic lot scheduling problem

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Abstract The economic lot scheduling problem has driven considerable amount of research. The problem is NP-hard and recent research is focused on finding heuristic solutions rather than searching for optimal solutions. This paper introduces a heuristic method using a tabu search algorithm to solve the economic lot scheduling problem. Diversification and intensification schemes are employed to improve the efficiency of the proposed Tabu search algorithm. Experimental design is conducted to determine the best operating parameters for the Tabu search. Results show that the tabu search algorithm proposed in this paper outperforms two well known benchmark algorithms.

Keywords Economic lot scheduling problem · Tabu search · Neighborhood search and design of experiment

1 Introduction

The Economic Lot Scheduling Problem (ELSP) deals with the production assignment of several products sharing the same production facility. The objective of the ELSP is to minimize total production cost. It is a constrained optimization problem in which production scheduling is done in such a way that all products are manufactured and their demands satisfied during the planning period. The production scheduling decision includes the determination of the production sequence and cycle time of each product in the sequence. A variety of methods have been proposed to solve the ELSP. In general, these methodologies can be categorized

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as: *common cycle approach*, *basic period approach* and *time varying lot size approach*. The common cycle approach is the simplest to implement when all products are manufactured in the same period. The basic period approach allows different cycle times for different products; however the cycle time of each product has to be an integer multiple of the basic period. Research shows that both of these approaches have limitations. The common cycle approach may produce solutions that are far from the Lower Bound (LB). The basic period approach tends to find better solutions than the common period approach; however finding a feasible solution is NP-hard (Hsu 1983). On the other hand, the time varying lot size approach is more flexible in solving the ELSP than the other two approaches as it allows different cycle times for products. Maxwell (1964) and Delporte and Thomas (1978) started using a time varying lot size approach to solve the ELSP. Dobson (1987) showed that under this approach any production sequence can be converted into a feasible sequence and proposed a solution known as Dobson's (1987) Heuristic (DH).

Although significant progress has been made in solving the ELSP, there is no reported method that can optimally solve large size scheduling problems within a desirable time frame. Recent efforts in this area have focused on the development of heuristics to find near optimal solutions. The performance of a heuristic method is measured by estimating its deviation from a lower bound, or by estimating improvements made over other existing algorithms.

A heuristic approach, Tabu Search (TS), is found to be a promising tool for solving NP-hard optimization problems (Glover and Laguna, 1998). It has the ability to reach the near optimal solution while escaping from the local minimum. In this study we propose a TS algorithm to solve the ELSP by using a time-varying approach. Our results demonstrate that TS algorithm produces better solutions to the ELSP then the two other benchmark methods. In the following section, a short literature review pertinent to the ELSP is presented. Section 3 outlines the ELSP model under a time-varying approach. Section 4 discusses the implementation of the TS algorithm to the ELSP and its parameter selection. A computational study to show the performance of the TS algorithm is presented in Section 5. Finally, conclusions and future research are described in Section 6.

2 Literature review

During the last four decades a large number of research tackling the ELSP has been proposed. Earlier works in the area are due to Eilon (1959), Hanssmann (1962), Rogers (1958) and Maxwell (1964) focused on finding the LB by using the Independent Solution (IS) approach. In the IS approach, the machine sharing constraints are ignored. An improved LB approach was developed by Bomberger (1966) by applying Karush Kuhn Tucker (KKT) conditions to the ELSP. Later works in the ELSP have focused on finding feasible cyclic schedules. A cyclic schedule is consistent with the Zero-Switch-Rule (ZSR). Using the ZSR, an item will be produced only if its inventory depletes to zero level.

Jones and Inman (1989) and Gallego (1990) showed that under certain conditions the common cycle approach generates very good solutions to the ELSP. As mentioned in the previous section, the basic period approach tends to perform better than the common cycle approach. Therefore, most of the heuristics are developed based on the basic period approach. This approach first selects a frequency for each item and then finds a feasible schedule that satisfies these frequencies. The earlier work using this approach is due to Bomberger (1966) and Doll and Whybark (1973). Elmaghraby (1978) provided a comprehensive review in this area covering the research up to the late 1970s. Hsu (1983) showed that using a basic period approach is NP-hard in determining a feasible schedule. Moreover, the complexity increases when the utilization ratio increases. The time-varying lot sizes approach does not consider converting a production sequence to a feasible sequence. Gallego and Shaw (1997) showed that the time-varying lot size approach to the ELSP is NP-hard in general. Dobson's (1987) heuristic can be integrated with Zipkin's (1991) algorithm to find a near optimal schedule for the ELSP under the time-varying lot size approach (Moon et al., 2002). The near optimal schedule enables one to determine both the production and setup times of a production sequence. Several extensions to the time-varying lot size approach have been reported in the literature. Dobson (1992) considered the problem when the setup time is sequence dependent and solved the problem using a heuristic approach. Wagner and Davis (2002) studied sequence dependent setup times and proposed a heuristic procedure that is able to produce multiple optimal solutions. The heuristic proposed by Wagner and Davis (2002) is preferable in precarious manufacturing environments. The ELSP is studied by Gallego and Roundy (1992) when back orders are in consideration.

Silver (1993) summarized several extensions to the existing quantitative models that make managerial decision making processes more meaningful. He pointed out that commonly known parameters to the model may be unknown in real life problems. He also described some improvements that can be incorporated into manufacturing systems and to the ELSP. Some of these are setup time/cost reduction, quality improvement, controllable production rates, lead time reduction, etc. A variety of modifications is documented by Silver et al. (1998). The effect of varying production rates to the ELSP were studied by Silver (1990), Moon et al. (1991), Gallego (1993), Khouja (1997), and Moon and Christy (1998). These works suggest that slowing the production rate of an under utilized facility is profitable. Silver (1995), and Viswanathan and Goyal (1997) considered the constraint of shelf life on a single production facility producing multiple products in repetitive cycles. Allen (1990) developed a graphical method for production rate and cycle time for a production facility producing only two products. Faaland et al. (2004) showed that allowing lost sales in the ELSP may generate higher profits. A computational study supports that lost sale could be profitable even with deterministic production and demand rates.

Gellego and Moon (1992) studied the trade-off between the setup time and the setup cost. They considered a factory producing multiple items and assumed that setup times can be reduced by externalizing setup operations with an increase in setup cost. They showed that a significant improvement is possible for highly utilized production facilities when such an approach is adopted. Moon (1994) considered setup time reduction on the ELSP by one time investment. Hwang et al. (1993) and Moon (1994) developed an ELSP model while considering setup time reduction and quality improvement through additional investments. Bourland and Yano (1997) studied three widely used approaches to a fixed-sequence ELSP. They considered the ZSR and Equal Lot (EL) and combination of both constraints in the formulation. Moon et al., (1998) considered stabilization of production rate on the ELSP. Moon et al. (2002) also considered an unreliable production facility in the ELSP where production starts with an "In-Control" state and could lead to an "Out-of-Control" state resulting in the production of non conforming items. In a recent work, Giri et al. (2003) included preventive and corrective maintenance and their costs into the model.

In a survey article, Silver (2004) discussed a wide range of heuristic methods that can be of interest to researchers and practitioners. Aytug et al. (2003) surveyed the use of Genetic Algorithms in production and operation management. Khouja et al. (1998) successfully applied the Genetic Algorithm to the ELSP while considering the basic period approach. In their work, deviation from the LB is observed to be 85% when machine utilization is high. Ouenniche and Boctor (1998) developed a non-linear mixed integer model for sequencing and lot sizing in job shop environment. A common cycle approach is used in this study. Only small size problems are found tractable. For medium and large size problems, Tabu Search and Simulated Annealing approaches are adopted. The present study also demonstrates an implementation of Tabu Search to solve the ELSP, however this work can be distinguish from Ouenniche and Boctor's (1998) work. In this study Tabu Search is used to solve the ELSP using time varying lot size approach at higher machine utilization. A common cycle approach results poor solution at higher machine utilization when it is compared with LB (Moon et al., 2002). Moon et al. (2002) proposed a Hybrid Genetic Algorithm (GA) to the ELSP by using the time varying lot size approach. Their method is able to outperform Dobson's (1987) heuristic, the best known heuristic method. In this paper we present also two Neighborhood Search heuristics to solve the ELSP efficiently using time-varying lot size approach. The proposed methods produced significantly improved results to the ELSP when compared to the DH and the Hybrid GA.

3 The ELSP model

The time varying lot size approach was developed by Dobson (1987) to solve the ELSP. Some specific ELSP assumptions are stated below.

1. Items (products) do not have any precedence over each other. They compete for the same production facility.
2. Backorders are not allowed.
3. Production facility is assumed to be failure free and always produces perfect quality products.

Following notations are common in literature for the ELSP problem:

- i: Product index
- j: Position index of an item in the schedule
- m: Total number of products
- n: Total number of production runs in a cycle
- p_i: Production rate of the item i, $\forall\, i = 1, 2, \ldots, m$
- d_i: Demand rate of the item i, $\forall\, i = 1, 2, \ldots, m$
- h_i: Inventory holding cost (\$ per unit per day), $\forall\, i = 1, 2, 3, \ldots, m$
- A_i: Set-up cost (\$) $\forall\, i = 1, 2, 3, \ldots, m$
- t^j: Production time for a product produced at the position j in the production sequence
- s_i: Set up time for the product i. (days) $\forall\, i = 1, 2, 3, \ldots, m$
- u^j: Machine idle time associated with the product processed at the jth position in the production sequence
- f^j: An item produced in the jth position
- $\mathcal{T}$: Length of the production cycle (in days)
- $K = 1 - \sum_{i=1}^{m} \frac{d_i}{p_i}$ (machine idleness)

In a typical production sequence $\mathbf{f}$ having n production runs such that $\mathbf{f} = \{f^1, f^2, \ldots, f^n\}, \forall f^j \in \{1, 2, 3, \ldots, m\}$, f^j contains an item $i \in \{1, 2, 3, \ldots, m\}$. We denote the production rate and demand rate of that particular item i produced at jth position in $\mathbf{f}$ with p^j and d^j respectively. Similarly the associated production and idle times are t^j and u^j respectively. The problem is to determine the optimal parameters for cycle length $\mathcal{T}$, a production sequence **f**, production times $\mathbf{t} = \{t^1, t^2, \ldots, t^n\}$ and idle times $\mathbf{u} = \{u^1, u^2, \ldots, u^n\}$, such that the demand is satisfied and the total setup and inventory holding costs are minimized. The cycle is repeated indefinitely. Let's consider the i^{th} product which is produced at the jth position in the production sequence. Its production involves a production time of t^j, a setup time of s^j and an idle time of u^j. The considered part will not be produced again until the remaining products are produced. The total number of parts produced in the jth position is $p^j t^j$. These parts will satisfy the demand for the product in period $[0, v]$, where $v = \frac{p^j t^j}{d^j}$. The highest inventory level is $(p^j - d^j)t^j$. The total inventory holding cost of the product that is produced at the jth position in the sequence is $1/2h^j(p^j - d^j)(p^j/d^j)(t^j)^2$. Let L_k represent the set containing products that are produced in a given sequence from k to the position in the sequence where the product k is produced again but not included in the same cycle. The ELSP problem that minimizes the total cost can be formulated as:

$$\inf_{\substack{\min \\ t \geq 0 \\ u \geq 0 \\ \mathcal{T} \geq 0}} \frac{1}{\mathcal{T}} \left(\sum_{j=1}^{n} \frac{1}{2} h^j (p^j - d^j) \left(\frac{p^j}{d^j} \right) (t^j)^2 + \sum_{j=1}^{n} A^j \right) \tag{1}$$

subject to

$$\sum_{j \in J_i} p_i t^j = d_i \mathcal{T} \quad i = 1, 2, \ldots, m \tag{2}$$

$$\sum_{j \in L_k} (t^j + s^j + u^j) = \left(\frac{p^k}{d^k} \right) t^k \quad k = 1, 2, \ldots, n \tag{3}$$

$$\sum_{j=1}^{n} (t^j + s^j + u^j) = \mathcal{T} \tag{4}$$

The first constraint, Eq. (2), states that the satisfactory amount of item i should be produced to fulfill its demand with a rate of d_i over a cycle time of $\mathcal{T}$. Equation (3) constrains the production rate of the item i, hence its demand is satisfied until it is produced again in the next cycle. Equation (4) indicates that the sum of production, setup and idle time of the product produced in any given sequence is equal to the cycle time $\mathcal{T}$.

4 Solution methods to ELSP

4.1 Tabu search algorithm

Glover (1989, 1990) introduced the Tabu Search (TS) algorithm. It is an iterative heuristic for solving combinatorial optimization problems such as the ELSP. The TS is a generalization of a local search. At each step, the local neighborhood of a current solution is explored and the best solution in that neighborhood is selected as the new current solution. Unlike the local search, which stops when no improved solution is found in the current neighborhood, TS continues to explore new solutions even if the new ones are worse than the current best solution. To prevent cycling, the information pertaining to the most recently visited solution is recorded in a list called *Tabu list*. The tabu status of a solution is overridden when the *aspiration criterion* is satisfied. A detailed discussion on the basic structure of the TS algorithm can be found in Glover et al. (1998) and Sait and Youssef (1999). The performance of the TS algorithm can be improved by the incorporation of some additional features like the search intensification and diversification (Glover et al., 1998; Ben-daya and Al-Fawzan, 1998). In the following sections we discuss the features and control parameters of the TS algorithm designed for solving the ELSP:

Seed solution

In our implementation, the seed solution is a feasible production sequence **f**, generated randomly by using production frequencies. Only feasible seeds are accepted in this search. A seed is considered to be feasible if it does not have the same products (items) at two adjacent positions and also does not contain the same items at the start and at the end of a production sequence **f**.

Neighborhood

A neighborhood production sequence **f′** for a given production sequence **f** can be generated from several schemes (Tillard, 1990). In this research, neighborhoods are randomly generated by exchanging an item in the ith position with another one in the kth position in a given **f**.

Tabu restriction

This is a control logic used to avoid cycling back to previously visited solutions. This is achieved by moving the selected attributes of an alternative solution to *Tabu* (forbidden). Attributes of a solution are recorded in the tabu list. In our implementation, attributes of a randomly generated neighbor production sequence **f** are the two randomly selected positions in **f** that result in **f′**.

Aspiration criterion

The aspiration criterion is used to override the tabu status of a solution when a new solution is better than the current best solution.

Search intensification and diversification

Search intensification is employed to explore the regions of good solutions in the search space with more intensity. Revisits to the previously explored neighborhoods are conducted to avoid the possibility of missing any good solutions due to the temporal locking. Search diversification is just the opposite of intensification. The search changes its direction to a new region that has not yet been explored by the algorithm.

Intensification criterion in this implementation uses frequency-based memory (Glover et al., 1998; Ben-daya and Al-Fawzan, 1998). In the proposed TS algorithm, an $m \times n$ frequency matrix $\mathbf{H}_{\mathbf{m}\times\mathbf{n}}$ whose elements are positions of the items in the production sequence is used. Products are listed in the rows of the matrix and their production schedules are listed in the columns. The entry H_{ij} is incremented by 1 if the item i is scheduled at the position j in the production schedule **f**. When the algorithm fails to observe further improvements within a predetermined number of iterations, the search intensification scheme (see Fig. 1)

```
Procedure Intensification
    (* m = Product index. *)
    (* n = Position index in the sequence *)
    (* H_{m×n} = Frequency matrix *)
    (* f = Production sequence *)
    (* z_i = Production frequency of item i *)
Begin
    f ← {∅}, k ← 1, j ← 1
    Repeat
        Find the entry in frequency matrix H such that H_ij = max{H}
        If (z_i ≥ 1) Then
            Insert Product i at the j^th position in the f
            z_i ← z_i − 1
        Endif
        Delete the i^th row and and column j^th from H
        k ← k + 1
    Until (k ≤ n)
End
```

Fig. 1 Intensification scheme

is triggered. The frequency matrix **H** is then used to generate a production sequence **f** and restart the TS by using **f**.

4.1.1 Implementation of the TS algorithm to the ELSP

In our implementation, the ELSP is solved by using the TS algorithm in the following four steps:

- **Step 1:** Determine the LB on the optimal cycle length T_i^* for each item using the LB computation procedure given in Bomberger (1966) and Moon et al. (2002).

- **Step 2:** Assuming T_i^* is the optimal cycle length for the item i, the optimal production frequency z_i is determined as shown below:

$$z_i = \frac{\max_j \{T_j\}}{T_i} \quad \forall\, i = 1, 2, 3, \ldots, m. \tag{5}$$

- **Step 3:** Round the production frequencies, found in Step 2, to the nearest integer.

- **Step 4:** The best production sequence is determined from the production frequencies found in Step 3. The cost of any given production sequence **f** is determined using Eq. (1) which requires **f** and **t** to be known. Next, using the quick and dirty heuristic (Moon et al., 2002), production times **t** are determined. This heuristic assumes zero idle time $\mathbf{u} = 0$ (this approximation works well for highly utilized production facilities). For a given production sequence, **f** is solved for **t** by using Eq. (3).

Table 1 Levels of design of experiment

Factor	Label	Levels		
Candidate List Size (CLS)	A	3	25	50
Tabu List Size (TLS)	B	1	7	20
Intensification and diversification	C	No		Yes

Table 2 General factorial design response is % relative deviation from LB

Source	DF	Sum of squares	F	Prob. of larger F
A	2	1.40031	1.042	0.3552
B	2	0.00971	0.007	0.9928
C	1	0.03227	0.048	0.8268
A×B	4	0.02930	0.011	0.9998
A×C	2	0.04929	0.037	0.9640
B×C	2	0.01258	0.009	0.9907
A×B×C	4	0.01440	0.005	0.9999

Table 3 General factorial design response is CPU time

Source	DF	Sum of squares	F	Prob. of larger F
A	2	9586.34	1278.922	0.0000
B	2	803.86	107.244	0.0000
C	1	1.86	0.496	0.4823
A×B	4	48.84	3.258	0.0133
A×C	2	0.71	0.095	0.9095
B×C	2	8.53	1.138	0.3229
A×B×C	4	5.55	0.370	0.8296

Table 4 Production frequencies using rounding off the power of 2 algorithm

Example	Items m	Production frequencies $\{y_1, y_2, \ldots, y_m\}$: Round-off to power-of 2	Round off to nearest integer
Mallya	5	{2, 1, 4, 2, 1}	{2, 2, 3, 3, 1}
Bomberger	10	{1, 4, 4, 8, 4, 2, 2, 16, 4, 4}	{1, 4, 4, 7, 5, 2, 1, 12, 4, 2}

4.1.2 General factorial design for fine-tuning TS parameters

In order to improve the performance of the TS algorithm, statistical experiments based on a general factorial design are performed. Three important factors, Candidate List Size (CLS), Tabu List Size (TLS) and the option of using intensification and diversification schemes are considered. Each factor combination is tested with ten problems randomly generated using the parameters from the Set 1 in Table 8. Tables 2 and 3 show the results of ANOVA where the measured responses are the percentage relative deviations from the LB and CPU times in seconds. It is found that the performance of the TS algorithm does not deteriorate due to the variations in the TS parameters. The CPU time increases with the increase on both CLS and TLS. Since most of the problems are solved in less than sixty seconds, we decided not to consider CPU times in the parameter selection.

As a result of the statistical analysis, the following parameters are selected:

- CLS is 20 and it is the best among the candidate neighbors.
- TLS is 7 and it is updated using the first-in-first-out rule.
- Intensification and diversification schemes are employed. Intensification is invoked after 250 iterations when no improvement is observed in the current search. Similarly, the diversification is invoked at 500 iterations if no improvement is recorded.
- The algorithm stops at 1000 iterations.

Table 5 Comparison on Bomberger and Mallya's problem

		Existing solutions		Proposed solutions		
Problem type	LB	DH	GA	TS	NS_a	NS_b
Mallya	57.726	60.874	60.911	60.911	60.911	60.782
Mallya*	–	–	–	60.782	–	–
Bomberger	122.945	128.339	126.12	125.31	125.754	130.346

*TS algorithm uses production frequencies rounded off to the power of 2 Mallya's problem.

Table 6 Result of different heuristic algorithms on Mallya's problem

Heuristic	Details
DH	**f** = {3, 4, 3, 1, 2, 3, 4, 3, 1, 5}
	t = {2.5047, 16.07, 4.4432, 12.774, 15.743, 2.0777, 13.263, 3.5533, 12.32, 10.546}
TS	**f** = {2, 3, 5, 4, 1, 3, 2, 4, 3, 1, 4}
	t = {9.192, 5.615, 12.919, 11.607, 11.647, 3.412, 10.093, 11.596, 6.382, 19.094, 12.730}
NS_a	**f** = {3, 1, 4, 2, 3, 5, 4, 1, 3, 2, 4}
	t = {6.382, 19.094, 12.730, 9.192, 5.615, 12.919, 11.607, 11.647, 3.412, 10.093, 11.596}
NS_b	**f** = {2, 3, 1, 4, 3, 5, 1, 3, 4, 3}
	t = {15.743, 3.8749, 10.551, 14.329, 3.8914, 10.546, 14.543, 2.3425, 15.004, 2.4701}
TS*	**f** = {1, 4, 3, 5, 1, 3, 4, 3, 2, 3}
	t = {10.5512, 14.3294, 3.89142, 10.546, 14.5431, 2.34248, 15.0036, 2.47009, 15.7427, 3.87493}

4.2 Neighborhood search heuristic

A Neighborhood Search (NS) heuristic (French, 1982, Sait and Youssef, 1999) explores the neighborhood of a given solution. The NS is a greedy heuristic. It only accepts those solutions that are superior to the best solution visited so far. We tested two neighborhood search heuristics, NS_a and NS_b that differ in production frequency rounding off scheme. NS_a uses the production frequencies rounded off to the nearest integers. On the other hand, the NS_b rounds the production frequencies to the power of 2 (Roundy, 1989). The LB and quick and dirty heuristic, used in the TS algorithm, are also employed with NS_a and NS_b. Both heuristics use the same neighborhood scheme mentioned in Section 4.1 and stops after 1000 iterations.

5 Computational results

The proposed Tabu Search (TS) algorithm is coded using MATLAB. The initial experiments are performed with the test problems given in Mallya (1992) and Bomberger (1966). The production frequencies resulted using two distinct rounding off procedures are reported in Table 4. The total cost resulted using different methods is summarized in Table 5. A detailed performance comparison of existing and proposed heuristics on solving the problems given in Mallya and Bomberger is presented in Tables 6 and 7.

While solving Mallya's (1992) problem both TS and NS_a algorithms generated a solution with an average daily cost of \$60.911. We observed that TS produces better solutions to the problem when the rounding off algorithm (Roundy, 1989) is used to determine production frequencies. In this case the daily cost is reduced to \$60.782 for the same example. The NS_b

Table 7 Result of different heuristic algorithms on Bomberger's problem

Heuristic	Details
DH	**f** = {8, 4, 5, 8, 9, 8, 4, 10, 6, 8, 3, 2, 1, 8, 4, 5, 8, 9, 8, 4, 10, 7, 8, 3, 2, 8, 4, 5, 8, 9, 8, 4, 10, 6, 8, 3, 2, 8, 4, 5, 8, 9, 8, 4, 10, 7, 8, 3, 2}
	t = {35.8256, 62.313, 22.8308, 40.0922, 95.293, 37.4462, 61.6508, 13.9507, 13.5698, 40.1746 42.6083, 25.2726, 27.8604, 30.4963, 53.124, 19.1867, 34.6237, 82.1248, 30.5236, 48.4096 13.1926, 10.1493, 28.2923, 42.1434, 25.11, 32.3382, 56.4482, 20.298, 36.5595, 86.7862 33.3795, 51.2624, 13.8256, 14.2906, 29.3761, 43.702, 26.1615, 33.6866, 58.7276, 21.2655 38.316, 91.0158, 33.6701, 53.8305, 14.7519, 10.746, 31.6915, 47.5066, 27.9323}
TS	**f** = {4, 8, 5, 9, 8, 6, 7, 3, 2, 8, 4, 5, 10, 8, 9, 8, 4, 3, 8, 5, 2, 1, 4, 8, 9, 8, 4, 6, 5, 3, 8, 2, 10, 4, 8, 9, 5, 8, 4, 3, 2, 8}
	t = {93.1766, 36.0186, 14.2169, 70.7668, 39.1481, 12.2983, 17.6698, 36.6966, 25.4799, 39.3299 57.1912, 14.1954, 22.0708, 31.5664, 74.7627, 36.6643, 46.959, 40.4543, 46.3969, 14.6485 22.5563, 23.5598, 49.5853, 32.9021, 77.9791, 44.8312, 46.0915, 11.2615, 13.6737, 35.3029 40.6085, 18.7135, 25.0488, 53.1489, 38.4438, 76.8786, 13.9449, 37.2705, 30.8039, 36.3449 21.5995, 38.9538}
NS_a	**f** = {4, 2, 8, 5, 9, 6, 8, 4, 1, 3, 8, 2, 5, 4, 8, 9, 7, 8, 4, 3, 5, 2, 8, 10, 4, 6, 8, 9, 5, 8, 4, 3, 2, 8, 4, 8, 5, 9, 8, 10, 3, 8}
	t = {68.1266, 20.3381, 44.3999, 14.2791, 80.086, 10.551, 44.1094, 43.7623, 23.5598, 37.8943, 39.9535, 20.8636, 15.1851, 58.9104, 41.2746, 79.4703, 17.6698, 45.9338, 39.5632, 36.6152, 12.0651, 20.8661, 36.4121, 18.1524, 55.52, 13.0088, 36.6449, 73.3055, 13.1861, 37.678, 29.8507, 33.5974, 26.2814, 33.9898, 81.2233, 35.398, 15.9639, 67.5254, 29.2912, 28.9671, 40.6917, 37.0489}
NS_b	**f** = {8, 4, 2, 8, 10, 3, 5, 8, 4, 7, 8, 9, 8, 1, 4, 2, 8, 5, 3, 6, 8, 10, 4, 2, 8, 9, 8, 4, 8, 3, 5, 8, 10, 4, 8, 9, 8, 2, 7, 4, 8, 6, 3, 8, 5, 4, 8, 9, 10}
	t = {37.1454, 59.1409, 29.5561, 36.199, 18.409, 47.7908, 19.5933, 34.2828, 68.5239, 12.4048, 40.4191, 12.1963, 51.3413, 32.9175, 32.0596, 75.9503, 20.2406, 48.1149, 28.5767, 44.7419, 22.8214, 34.6986, 20.0756, 62.7295, 36.3913, 86.3812, 35.3577, 29.7402, 8.49047, 45.1615, 26.0659, 15.6664, 46.1008, 33.1044, 20.4657, 58.1256, 43.1214, 97.4225, 5.03985}

Table 8 Distribution for randomly generated data for the test problems

Parameters	Set 1	Set 2	Set 3
Number of items (units)	[5, 15]	[5, 15]	[5, 15]
Production rate (units/unit time)	[2000, 20000]	[4000, 20000]	[1500, 30000]
Demand rate (units/ unit time)	[1500, 2000]	[1000, 2000]	[500, 2000]
Setup time (time/ unit)	[1, 4]	[1, 4]	[1, 8]
Setup cost ($)	[50, 100]	[50, 100]	[10, 350]
Holding cost ($)	[1/240, 6/240]	[1/240, 5/240]	[5/240000, 5/240]
K	≤ 0.1	≤ 0.1	≤ 0.1

also finds the same cost for the problem. When the same problem set is solved by the GA given in Moon et al. (2002), a slight increase in the total cost is observed ($60.911) (see Tables 5 and 6).

Similarly, the results obtained for Bomberger's (1966) problem at 99% utilization are shown in Tables 5 and 7. The TS algorithm is able to find a better solution for the Bomberger's problem. NS_a is also able to find a better solution than DH and Hybrid GA.

Moon et al. (2002) tested the performance of GA using the Data Set 1 given in Table 8, while Dobson (1987) calibrated the performance of DH by using Data Sets 2 and 3. We generated fifty random problems for each data set given in Table 8 and studied the performance of

Table 9 Comparison of algorithms on randomly generated problems using Set 1

Parameters	Comparison with lower bound				Comparison with Dobson heuristic		
	$\frac{DH}{LB}$	$\frac{TS}{LB}$	$\frac{NS_a}{LB}$	$\frac{NS_b}{LB}$	$\frac{DH}{TS}$	$\frac{DH}{NS_a}$	$\frac{DH}{NS_b}$
Mean	1.0517	1.0242	1.0250	1.0484	1.0268	1.0260	1.0031
Min	1.0114	1.0074	1.0074	1.0114	0.9956	0.9919	0.9919
Max	1.2216	1.1100	1.1192	1.2176	1.1056	1.1055	1.0283
σ	0.0344	0.0183	0.0200	0.0328	–	–	–
Avg. CPU time (sec.)	–	–	–	–	11.0777	0.3828	0.4081
Best time (sec.)	–	–	–	–	3.2662	0.0288	0.0447
No. of problems with ratio $\leq$1	0	0	0	0	8	8	27

Table 10 Comparison of algorithms on randomly generated problems using Set 2

Parameters	Comparison with lower bound				Comparison with Dobson heuristic		
	$\frac{DH}{LB}$	$\frac{TS}{LB}$	$\frac{NS_a}{LB}$	$\frac{NS_b}{LB}$	$\frac{DH}{TS}$	$\frac{DH}{NS_a}$	$\frac{DH}{NS_b}$
Mean	1.0503	1.0272	1.0278	1.0491	1.0227	1.0220	1.0011
Min	1.0070	1.0051	1.0051	1.0070	0.9758	0.9722	0.9904
Max	1.2336	1.0708	1.0748	1.2054	1.2034	1.1967	1.0234
σ	0.0369	0.0136	0.0139	0.0330	–	–	–
Avg. CPU time (sec.)	–	–	–	–	11.3887	0.4297	0.4088
Best time (sec.)	–	–	–	–	4.5018	0.0528	0.0590
No. of problems with ratio $\leq$ 1	0	0	0	0	12	13	31

Table 11 Comparison of algorithms on randomly generated problems using Set 3

Parameters	Comparison with lower bound				Comparison with Dobson heuristic		
	$\frac{DH}{LB}$	$\frac{TS}{LB}$	$\frac{NS_a}{LB}$	$\frac{NS_b}{LB}$	$\frac{DH}{TS}$	$\frac{DH}{NS_a}$	$\frac{DH}{NS_b}$
Mean	1.2550	1.1592	1.1745	1.2301	1.0443	1.0311	1.0089
Min	1.0193	1.0107	1.0123	1.0192	0.9304	0.9304	0.9502
Max	8.1570	5.3720	5.4625	7.3938	1.5184	1.4933	1.1032
σ	1.0155	0.6206	0.6318	0.9054	–	–	–
Avg. CPU time (sec.)	–	–	–	–	22.1410	2.4722	5.0455
Best time (sec.)	–	–	–	–	14.5094	2.2650	4.8128
No. of problems with ratio $\leq$ 1	0	0	0	0	12	18	21

the proposed algorithm by comparing to the best known DH and GA. In their work, (Moon et al., 2002) reported an average of 1.1% improvement over the DH. The percentage average relative improvement of the proposed algorithms in our study (TS, NS_a and NS_b) over DH is 2.68 , 2.60 and 0.31 respectively. These results are obtained by solving 50 random problems generated from the Data Set 1. The performance of the proposed methods are also compared with the LB. Comparisons with the LB are presented in Table 9. The TS and NS_a are able to reduce the average relative deviation from LB by 2.42% and 2.50% respectively, while the average relative deviation from the LB of the GA (Moon et al., 2002) and the DH were 3.02% and 5.17% respectively. A similar study is conducted on problems generated from Data Sets 2 and 3 with parameters given in Table 8. The Data Set 3 contains more difficult problems (Dobson, 1987) and computational experience with this set is shown in Table 11. For the problems generated from the Data Set 3, the TS finds improved solutions over the DH on an average of 4.43%. For the same problem set, NS_a and NS_b generate solutions 3.11% and 0.89% better than the DH respectively. The standard error (σ) in the mean performance of TS is the least among all other heuristics we evaluated in this work. However, the CPU time required by the TS is slightly higher than the NS_a and NS_b. On average, the TS converges to the best solution within the first fifteen seconds when tested on the Data Set 3 problems.

6 Conclusions

In this paper three heuristic methods to solve the ELSP are proposed. The computational study revealed that the performance of the TS algorithm is superior to the best known Dobson's heuristics and Genetic algorithm. We used a simple pairwise neighborhood generation mechanism which is able to converge to the optimal solution with in a reasonable CPU time. The neighborhood generation mechanism is quite important for the performance of both the TS and the neighborhood search heuristics. The pairwise neighborhood generation mechanism seems to be the best choice as the performance of the TS remains consistent over a wide variety of problems. This mechanism also enables the NS_a to generate better solutions than the Dobson's heuristic and the Genetic algorithm. It is also obvious from the computational experiments that the NS_a performs better than the NS_b while both use the same operating parameters, except the rounding off procedure to determine the production frequencies of each item. From these results we conclude that the rounding off to the nearest integer results in a better solution to the ELSP. While testing Mallya's (1992) problem, we discovered that the proposed algorithms are able to determine multiple optimal solutions as well. These alternative solutions could be of interest in a precarious manufacturing environment. The improvement of TS over DH becomes considerable (approximately 4.43%) when tests are performed on harder problems.

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