Deconflicted Air-Traffic Planning With Speed-Dependent Fuel-Consumption Formulation

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***Abstract*—This paper discusses a unique formulation for the en-route flight planning problem in a constrained airspace with the objective to minimize costs incurred from earliness, lateness, and fuel-consumption and to ensure flight safety. Mid-air conflict and collision avoidance, minimum separation distance between aircraft and speed-dependent fuel-consumption-rate are explicitly formulated. A 3D mesh network consisting of waypoints is used to provide alternative routing options for aircraft. The formulation of fuel-consumption-rate as a function of speed as part of the air-traffic planning (ATP) problem is unique in the literature. Moreover, this paper is the first attempt to model the mid-air conflict and collision avoidance as part of the ATP problem. In order to demonstrate the capabilities of the mathematical model, test instances were generated and solved by three different solution strategies. The proposed centralized solution strategy can optimally solve small size instances, similar to the air-traffic around airports to help air-traffic control authorities to manage arrival and departure sequences. Larger networks that include several airports can be solved by the proposed two sequential solution strategies (decentralized and hybrid solution strategies) to help air-traffic planning authorities to manage air-traffic safely and more economically.**

***Index Terms*—Air-traffic control, air-traffic flow management, fuel optimal control, conflict and collision avoidance, speed-dependent fuel-consumption.**

I. INTRODUCTION

DEMAND for airline services has been steadily increasing around the world. Around the world, both the seat capacity and the number of airline companies have increased significantly. While long-haul routes are still dominated by the major carriers such as United Airlines, Lufthansa or Singapore Airlines, relatively smaller airline companies have gained important market shares in the short-flight markets. These new market conditions have brought many challenges as well as benefits for the industry. Crowded airports and airspace, volatile fuel prices, increasing environmental awareness and labor costs, and unpredictable weather conditions in most parts of the world are challenging many airline companies. The transportation authorities, in particular Air-Traffic COntrollers (ATCOs) are also impacted from the current market conditions. From taxing to facilitating safe navigation in open skies, ATCOs play a crucial role in delivering on-time services and ensuring the safety of aircraft and passengers. During an aircraft's journey, its speed and route are planned by the airline, and approved and monitored by the responsible Air-Traffic Control (ATC) authorities. Pilot's discretion in the en-route flight planning process is rarely an option (pilots make real-time decisions in emergency situations). FAA anticipates an increase of 56.9% in control tower operations and over 100% in en-route (high altitude flights) traffic-control operations by 2030 [1]. It is clear that increasing air-traffic is undermining ATCOs' ability to effectively manage the given flight plans. Long working hours, stressful working conditions and continuously increasing air-traffic volume may lead to poor decision making by ATCOs, necessitating an increasing reliance on tactical collision avoidance systems embedded in airplanes to avoid mid-air collisions [23]. Furthermore, due to high air-traffic volume, safety issues must be addressed before the lower priority issues like economic and service objectives of airlines can be accommodated.

Recently, an alternative Air-Traffic Flow Management (ATFM) strategy, namely Free Flight Concept (FFC) has been proposed to reduce the workload of ATCOs and improve the air-traffic flow. FFC aims at transferring the en-route flight planning task to individual aircraft ([14], [19], and [32]). The motivation for the mathematical model introduced in this paper came from the FFC philosophy. The goal is to assist both pilots and airline companies to create safe and economic flight plans and assist ATCOs to make real-time decisions for rerouting aircraft safely and at the same time still considering the business objective of the companies. In today's air-traffic planning, an aircraft completes its journey from an origin to a destination by visiting a set of waypoints which are geographical coordinates in the sky. Hence, we propose a mathematical model that navigates aircraft through these waypoints between origin and destination airports. The waypoint based mixed-integer linear programming (MILP) model takes all meaningful airport and flight characteristics into consideration to provide a comprehensive and thorough model of the situation. The produced flight plan for each aircraft includes the sequence of waypoints to be visited, the aircraft's exact arrival and departures times at these waypoints, the average speed between consecutive

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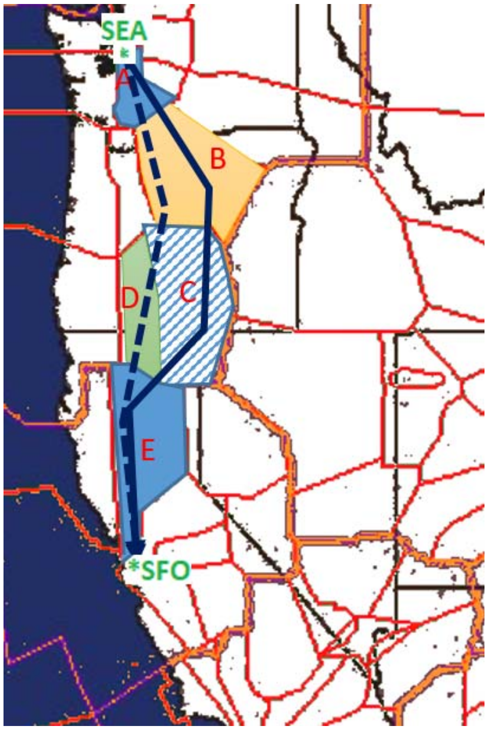


Fig. 1. Entrance and exit locations to/from the air sector impacts flight time in the air sector: Flight from Seattle to San Francisco.

waypoints and the fuel-consumption rates at that speed. While the objective is to minimize the earliness, tardiness and the fuel-consumption costs, the mid-air conflict avoidance is explicitly handled. Despite its computational complexity in comparison to alternatives available in the literature (e.g. [9]), benefits of the proposed mathematical model are crucial for the future of the air transportation. Potential benefits are:

- Increased airspace utilization: Even around the densely used airspace, the proposed waypoint based formulation provides significant flexibility for individual aircraft to perform collision-free navigation.
- Fuel-consumption: The fuel-consumption-rate as a function of speed is formulated without losing the linearity of the model. Consequently, more environmental friendly and less costly air traveling is made possible.
- Accurate traveling time: Most air sector based ATFM formulations in the literature assume that flight duration in a given air sector can be bounded by a minimum and maximum flight time. In reality on the other hand, the flight time in an air sector is strictly depends on the entrance and exit points at the air sector. As illustrated in Fig. 1, flight time in a sector would significantly vary depending on the route taken. In the proposed waypoint based formulation, we successfully addressed this problem by determining more accurate flight duration in each sector.
- Address complexity and assure accuracy: Three strategies (solution methods) are proposed. i) Centralized solution strategy where flight plans for all incoming and departing aircraft are determined at the beginning of the planning horizon; ii) Sequential (decentralized) aircraft management strategy which conforms very well with the objectives of the NASA's FFC (aims at giving the autonomy to the pilots in en-route flight planning [20]) where each airplane determines its flight plan with respect to the given current traffic conditions; iii) A Hybrid solution strategy, that combines both centralized and decentralized approaches, overcomes the scalability issue of the centralized approach and provides much better results than the decentralized approach.

The remainder of the paper is organized as follows. In Section II, a brief literature review, in Section III model, and in Section IV solution and results are introduced. Finally in Section V, the conclusions and the plans for future work are summarized.

II. LITERATURE REVIEW

In this section we discuss the relevant literature on ATC and ATFM problems. While there exists a rich literature on ATFM problems, we only cover topics that are directly related to the scope of our paper. For more general discussion on the ATFM problem, readers are referred to the works of Navazio and Romanin-Jacur [29] and de Neufville *et al.* [30]. Studies in airport and airspace congestion, mid-air conflict resolution, dynamic speed control and finally the speed-dependent fuel-consumption problems are discussed below.

A. Air-Traffic Control and Flow Management

Earlier works on ATFM focus on ATC and airport congestion problems. Airport congestion problems are further categorized as aircraft landing and take-off problems [8]. The research on ATC on the other hand mostly focuses on the technological innovations. Only a handful of operations management literature that focuses on determining and improving the size and capacity of air sectors with the objective of improving the overall performance of ATC are available ([22], [24], [28], and [38]).

More recent works in ATFM area focus on the determination of an economically sound en-route flight plan without causing congestion in the air sectors and around airports. Strategies such as ground and airborne delays/holdings are crucial for the air sector capacity management. Odoni [31] was one of the earliest to study the ground holding strategies for a single airport. Later, dynamic ground holding problem in a single airport is introduced [5], [21], [25], [27], and [37]. As the demand for air-travelling has increased, a new line of research for developing ground and airborne holding strategies for multi-airport networks has emerged [2], [15], and [36]. In their review paper, Navazio and Romanin-Jacur [29] summarized the works on multi-airport ground holding problems. It is clear that the airborne delays are more expensive than the ground holding costs. Yet, at the operational level, airborne delays are necessary to absorb the impacts of unexpected weather and air-traffic conditions. Hence, when needed, airborne delays should be handled with the least expensive ways. Consequently, the research focus has shifted on the re-routing strategies to minimize the impact of airborne delays. The rerouting concept has shifted the research focus on the ATFM problem. Consequently, the congestion problem is tackled in the entire airspace rather than at a single airport. Helm [18] introduced one of the earliest rerouting formulations. Bertsimas and Patterson [11] and later Dell'Olmo and Lulli [14] developed mathematical models for enabling rerouting to respond changing traffic and weather conditions. Other notable works on the rerouting problem are Bertsimas *et al.* [9], Leal de Matos *et al.* [13], and Ma *et al.* [26].

B. Fuel Consumption

The fuel-consumption problem in ATFM is relatively new. Most researches have focused on developing technologies to build more fuel efficient aircraft designs [7]. Regardless of the technology being used, it is known that the aircraft fuel-consumption-rate varies depending on the flight speed. Fuel-consumption is not only a cost issue. Aircraft emissions have been a major contributor to air quality, particularly around airports. However, due to the complex nature of the en-route flight planning problem, the aircraft fuel-consumption issue as part of the ATFM problem has not been studied. One of the most notable works that studies the relationship between the fuel-consumption and the speed of an aircraft is the work of Clarke *et al.* [12]. More recently, Vela et al. [35] proposed a model for conflict resolution while ensuring the optimal fuel-consumption rate. None of these works treats the fuel-consumption problem as part of the ATFM problem.

C. Mid-Air Collision Avoidance

In recent years, collision avoidance has been studied from the operations planning perspective where automated collision free path planning tools have been introduced [3], [4], [17], [33]. In their review article, Kushar and Yang [6] compare 68 conflict detection and resolutions methods using 5 different criteria: State Propagation; State Dimensions; Conflict Detection; Conflict Resolution; and Resolution Maneuvers. While some of the methods reviewed by Kushar and Yang are currently being tested and used in the industry, none of the 68 methods reviewed provides a reliable and effective solution to automate the conflict detection and resolution process in the aviation industry. Moreover, the most literature focuses on the safety aspect alone. Operational expectations such as minimization of delays and fuel-consumption are not well integrated in the conflict detection and resolution literature. The proposed mathematical model in this paper formulates the collision avoidance as part of the ATFM problem. However, the collision avoidance is only guaranteed at the waypoints. At the operational level, the proposed model must be supported by the conflict detection and resolutions methods similar to the ones discussed in Kushar and Yang [6] in order to guarantee the required separation between flights on two consecutive waypoints. Since the model described in this paper avoids conflict at the nodes, handling of conflict avoidance between waypoints is trivial.

D. Comparison With the Current Literature

The major contributions of the paper that are unique in the literature are:

- The model introduces time as a decision variable rather than periods where decisions are made only at the beginning of each period. In the current ATFM literature, state-time network is used in the formulation where $t = \{0, 1, \ldots, T\}$ is a period, and arrival and departure of an aircraft at an airspace occur in one of the predefined periods ([9] and [10]). In such formulations, the continuous notion of time is ignored.
- The formulation allows aircraft to modify its speed during flight.
- The fuel-consumption-rate as a function of speed is embedded in the ATP formulation. To the best of authors' knowledge, this is the first attempt to link fuel-consumption-rate with the aircraft speed as part of the ATP problem.
- The collision avoidance and separation distance concepts are explicitly formulated.
- The waypoint based formulation captures the real-time traffic conditions more accurately than the sector based formulations. The assumption which is common in the sector-based ATFM formulations for estimating the flight duration with a predefined bound without knowing the exact entrance and exit locations is not accurate. An aircraft's traveling time in the airspace varies significantly depending on its entrance and exit locations. Let us consider two alternative routes for an aircraft traveling from Seattle to San Francisco. As illustrated in the high altitude air route traffic control center map (Fig. 1), even though both alternative routes follow the highlighted sectors A, B, C, D, and E in the flight plans, the traveling time particularly at sectors C, D would significantly differ due to different entrance and exit locations.
- Waypoint-based en-route flight planning models are computationally more challenging in comparison to the air sector-based studies such as the one introduced in [9]. Yet, advantages such as better utilized airspace, conflict resolutions and the optimized fuel usage characteristics are sufficient to justify the additional complexity. Furthermore, decentralized and hybrid solution strategies are introduced to tackle the complexity problem.

III. PROBLEM FORMULATION

The en-route flight planning problem is formulated in a way that, all flights enter a 3D mesh network (see Fig. 2 for illustration) from one of the available entrance waypoints (v^f_{IN}) and they travel by visiting transition waypoints (v) through links (ℓ) to reach their destinations (v^f_{OUT}) on time with a minimum cost and without violating safety rules. The mathematical formulation determines a flight plan (R^f) that includes a set of links ($x^f_\ell = 1$), arrival (a^f_ℓ) and departure (d^f_ℓ) times, average speed (s^f_ℓ) and the fuel-consumption-rate (FCR^f_ℓ) on these links as $R^f = (x^f_\ell, a^f_\ell, d^f_\ell, s^f_\ell, \mathrm{FCR}^f_\ell, \forall \ell \in L : x^f_\ell = 1)$.

The details of the model are discussed in the following three subsections. First, we provide the list of parameters and decision variables. Second, the assumptions made in the modeling are summarized. Finally, the problem formulation is introduced.

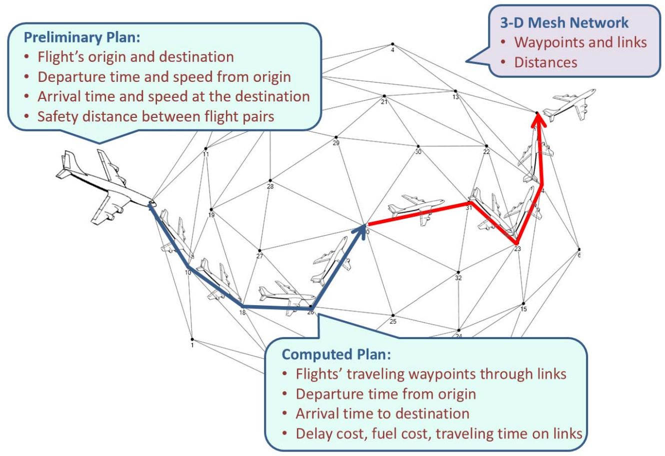


Fig. 2. Illustration of 3D mesh network used in the formulation.

A. Model Parameters and Decision Variables

1) Parameters: The definition of parameters are provided below:

F is a set of flights, indexed by f

V^+ is a set of all waypoints including entrance and exit waypoints, indexed by υ

V^- is a set of transition waypoints (entrance and exit waypoints are not included)

L is a set of links, indexed by ℓ

υ_{IN}^f is the entry node for flight f

υ_{OUT}^f is the exit node for flight f

LENGTH_ℓ is the distance between two connected waypoints

$\omega_\ell^-(\upsilon)$ is a set of allowed incoming links for a flight leaving υ through link ℓ

$\omega_\ell^+(\upsilon)$ is a set of allowed outgoing links for a flight entering υ through link ℓ

The definition of ω is illustrated in Fig. 3.

t_{IN}^f and t_{OUT}^f are scheduled arrival and departure times

$t^{ff'}$ is the required separation distance (expressed in time units) for flight f' following flight f

$\tau^{\min}$ is the time to travel a unit distance with the minimum possible speed

$\tau^{\max}$ is the time to travel a unit distance with the maximum possible speed

P_{EARLY}^f and P_{LATE}^f are the penalty costs for early/late arrivals

FCOST_U is the per gallon fuel cost

In this paper, we demonstrate the feasibility of using two different speed control strategies: discrete; and continuous. Yet, only the results for continuous speed, coupled with the fuel-consumption-rate are reported in the paper. Following parameters are used for the discrete speed control policy:

S is the set of possible speeds (discretized), indexed by s

Δ is the predefined set of speed changes between two consecutive links, indexed by δ

T is the set of possible traveling times when speed is discrete, indexed by t

2) Decision Variables: Below is the definition of decision variables.

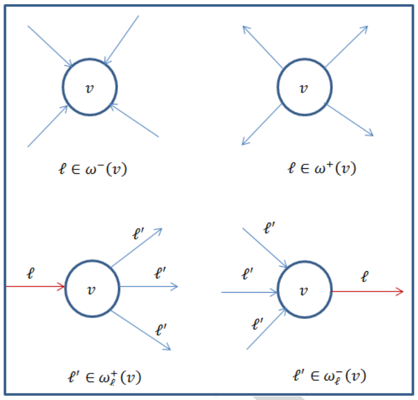


Fig. 3. Definition of parameter ω for different cases.

$$x_\ell^f = \begin{cases} 1 & \text{if flight } f \text{ travels on link } \ell \\ 0 & \text{Otherwise} \end{cases}$$

$a_\ell^f \in R^+$ is the arrival time for flight f to link $\ell \in \omega^+(\upsilon)$ from waypoint υ

$d_\ell^f \in R^+$ is the departure time for flight f from link $\ell \in \omega^+(\upsilon)$ that originates from waypoint υ

τ_ℓ^f is the time to travel a unit distance for flight f on link ℓ (utilized to derive actual speed)

$$\beta_\ell^{ff'} = \begin{cases} 1 & \text{flight } f' \text{ follows flight } f \text{ on link } \ell \\ 0 & \text{Otherwise} \end{cases}$$

$$\theta_\upsilon^{ff'} = \begin{cases} 1 & \text{flight } f \text{ leaves waypoint } \upsilon \text{ before} \\ & \text{flight } f' \\ 0 & \text{Otherwise} \end{cases}$$

$$\alpha_\ell^{ff'} = \begin{cases} 1 & f \text{ enters } \ell \text{ before } f' \\ & \quad \text{enters from the opposite direction} \\ 0 & \text{Otherwise} \end{cases}$$

The following decision variables are derived from the speed (τ_ℓ^f). They are independently defined to better describe the mathematical model.

FCR_ℓ^f is the fuel-consumption-rate for per unit of flight time

FCOST_ℓ^f the fuel-consumption cost in link ℓ

t_ℓ^f is the travel time for flight f on link ℓ

t_{EARLY}^f is the earliness of flight f

t_{LATE}^f is the lateness of flight f

$$x_{\ell s}^f = \begin{cases} 1 & \text{flight } f \text{ travels on } \ell \text{ with speed } s \\ 0 & \text{Otherwise} \end{cases}$$

$$y_{\ell\delta}^f = \begin{cases} 1 & \text{speed of } f \text{ is increased by } \delta \text{ on } \ell \\ 0 & \text{Otherwise} \end{cases}$$

s_ℓ^f is the speed of flight f on link ℓ

B. Assumptions

We made the following assumptions in order to realize the proposed flight planning model:

- An aircraft can visit a waypoint only once
- The speed change of an aircraft from link ℓ to the consecutive link ℓ' is bounded proportional to its current speed on link ℓ. This assumption is necessary to replicate the real flight conditions since the aircraft cannot change its speed drastically during flight

- The proposed MIP model determines only the average speed between two consecutive waypoints
- The fuel-consumption cost is determined based on the average speed
- The cost of per gallon jet fuel is assumed to be same for all types of aircraft

C. Problem Formulation

In order to solve a large scale optimization problem, it is important to obtain a strong formulation. The proposed formulation avoids non-linearity under all circumstances, yet still archives all its objectives. The described Mixed Integer Linear Programming (MILP) model considers the minimization of total cost that is incurred from delays, earliness and speed-dependent fuel-consumption. Constraints for the model are categorized in 4 groups: routing; timing; speed and fuel-consumption; and safety and conflict resolution.

1) Objective Function: Let us first define the objective function that is used in all case studies discussed later in section IV.

$$\min \sum_{f\in F} P^f_{\mathrm{EARLY}}\, t^f_{\mathrm{EARLY}} + P^f_{\mathrm{LATE}}\, t^f_{\mathrm{LATE}} + \sum_{\ell\in L} \mathrm{FCOST}^f_\ell \tag{1}$$

The cost incurred from earliness, tardiness and fuel-consumption during flight is minimized. The relationship between speed and fuel-consumption is discussed later in the paper. Cost of delays are the collection of airport penalties, additional fuel usage and labor cost (pilots and flight attendance). In 2015, it was estimated that the cost of per minute delay for airline companies is \$65.43 [16].

2) Flight Routing Constraints: Following constraints ensure that a given aircraft travels from its origin to the destination by traveling through available waypoints.

For all $f\in F$:

$$\sum_{\ell\in\omega(v)} x^f_\ell = 1, \quad v\in v^f_{\mathrm{IN}} \tag{2}$$

$$\sum_{\ell\in\omega(v)} x^f_\ell = 1, \quad v\in v^f_{\mathrm{OUT}} \tag{3}$$

$$\sum_{\ell\in\omega^-(v)} x^f_\ell = \sum_{\ell'\in\omega^+(v)} x^f_{\ell'}, \quad v\in V^- \tag{4}$$

$$x^f_\ell + \sum_{\ell'\in\omega^+_\ell(v)} x^f_{\ell'} \le 1, \quad \ell\in L \tag{5}$$

$$\sum_{\ell\in\omega^-(v)} x^f_\ell \le 1, \quad v\in V \tag{6}$$

$$\sum_{\ell\in\omega^+(v)} x^f_\ell \le 1, \quad v\in V \tag{7}$$

Constraints (2) and (3) ensure all flights depart from their origins and reach their destinations. Conservation constraint (4) forces all flights entering a transition waypoint to leave the waypoint. As illustrated earlier in Fig. 3, depending on the link that an aircraft arrives at a waypoint, there are only a limited set of links available for the aircraft to leave the waypoint. The constraint (5) is utilized to limit the aircraft's departure links. Inequalities (6) and (7) limit an aircraft to visit a waypoint and a link only once (Assumption 1).

3) Timing Constraints: Next, we introduce a set of constraints to control the relationship between arrival and departure times on waypoints and links.

For all $f\in F$:

$$x^f_\ell\, t^f_{\mathrm{IN}} \le a^f_\ell \tag{8}$$

$$a^f_\ell \le M x^f_\ell, \quad \ell\in L \tag{9}$$

$$d^f_\ell \le M x^f_\ell, \quad \ell\in L \tag{10}$$

$$\sum_{\ell\in\omega(v)^-} a^f_\ell = \sum_{\ell'\in\omega(v)^+} d^f_{\ell'}, v\in V\setminus\{v^f_{\mathrm{IN}}, v^f_{\mathrm{OUT}}\} \tag{11}$$

$$\sum_{\ell\in\omega(v^f_{\mathrm{OUT}})^-} d^f_\ell = t^f_{\mathrm{OUT}} + t^f_{\mathrm{LATE}} - t^f_{\mathrm{EARLY}} \tag{12}$$

Inequality (8) enforces aircraft to respect earliest departure times. Constraints (9) and (10) force arrival or departure times to be zero if the link is not used. In constraint (11), it is assured that the aircraft is not delayed at the intermediate waypoint. Finally, in Equation (12), exact earliness or tardiness is determined. In our case, arrival time at a waypoint is equivalent to the departure time from the connecting link (d^f_ℓ).

4) Speed Control Constraints: In the proposed mathematical model, the flight time between two consecutive waypoints is determined based on the flight speed. The distance between two consecutive waypoints (LENGTH_ℓ) is known. Therefore, the traveling time on a given link ℓ with an average speed (s^f_ℓ) is:

$$t^f_\ell = \mathrm{LENGTH}_\ell / s^f_\ell, \quad \ell\in L, f\in F \tag{13}$$

which is a nonlinear term. In order to avoid the non-linearity, two different speed control policies can be adopted: *(i)* Discrete speed control; and *(ii)* Continuous speed control in which the speed is substituted by the *time to travel a unit distance.*

- Discrete speed control constraints

Flight time as a function of speed can easily be derived by utilizing a discrete variable. In a given link ℓ with a known link length (LENGTH_ℓ), for any speed in the speed set ($s\in S = \{s_1,\ldots,s_n\}$) there exists a corresponding flight time as $t_{\ell s}\in \mathrm{T}_\ell$. A binary decision variable $x^f_{\ell s}$ is utilized to connect current speed with the flight duration. Consequently,

$$d^f_\ell = a^f_\ell + \sum_{s\in S(s^f_\ell)} x^f_{\ell s} t_{\ell s} \quad \ell\in L, \ f\in F \tag{14}$$

is derived. While discrete speed control is easier to model, segmentation of speed increases the computational complexity and reduces the accuracy. Consequently, a continuous speed control policy is formulated.

- Continuous speed control constraints

Let τ_ℓ be the required time to fly a unit distance with a given speed on link ℓ, where $\tau_\ell = 1 / s_\ell$. Describing speed in terms of time to travel a unit distance enables us to determine the traveling time on a link with a linear expression as:

$$t^f_\ell = \tau^f_\ell\, \mathrm{LENGTH}_\ell, \quad \ell\in L, \ f\in F \tag{15}$$

Consecutively:

$$d^f_\ell = a^f_\ell + \tau^f_\ell\, \mathrm{LENGTH}_\ell, \ \ell\in L, \ f\in F \tag{16}$$

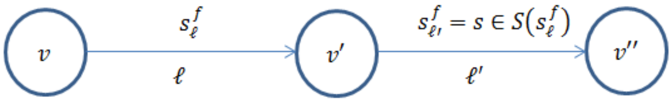


Fig. 4. Speed change from one link to the other.

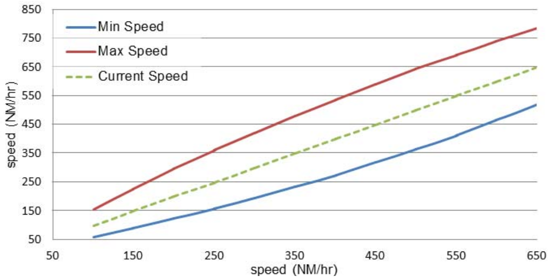


Fig. 5. Speed control between consecutive links (speed is expressed in Nautical Mile - NM per hour).

Assume that the speed on the consecutive link depends on aircraft's current speed (see Fig. 4 for illustration). Hence, a relationship similar to the Equation (17) is required to bound the speed changes from the current link to the next one.

$$(1-p_1)\tau^f_{\ell'} \le \tau^f_\ell \le (1+p_2)\tau^f_{\ell'} \quad \ell \in L,\ \ell' \in \omega^-_\ell(v),\ f \in F \tag{17}$$

However, constant speed change parameters (p_1 and p_2: allowable speed reduction and increase rates) may lead to significant speed changes from one link to the consecutive one. In this paper, in order to sustain a smooth transition between two consecutive links, a proportional speed change policy is adopted (speed change from one link to the next link is bounded). A numerical study revealed that, Equation (18) along with Equations (19) and (20) would imitate the desired speed-control policies as illustrated in Fig. 5. By calibrating smoothing parameters $w^{\min}$ and $w^{\max}$, a variety of speed bounds can be generated as a function of current speed τ^f_ℓ.

$$\tau^f_\ell - \tau^{f-}_\ell \le \tau^f_{\ell'} \le \tau^f_\ell + \tau^{f+}_\ell,\ \ \ell \in L,\ \ f \in F \tag{18}$$

In Equation (18), speed increase τ^+_ℓ and speed decrease τ^-_ℓ limits are assumed to be flight specific and determined as:

$$\tau^{f-}_\ell = \underline{w}^{\min}(\tau^{\min} - \tau^f_\ell) + \underline{w}^{\max}(\tau^f_\ell - \tau^{\max}) \tag{19}$$

$$\tau^{f+}_\ell = \overline{w}^{\min}(\tau^{\min} - \tau^f_\ell) + \overline{w}^{\max}(\tau^f_\ell - \tau^{\max}) \tag{20}$$

In Fig. 5, we plot the speed s^f_ℓ on a unit distance. It is then bounded as follows:

$$\underline{s}^f_\ell = 1/(\tau^f_\ell + \tau^{f+}_\ell) \le s^f_\ell \le \overline{s}^f_\ell = 1/(\tau^f_\ell - \tau^{f-}_\ell)$$

Equations (19) and (20) are used to calculate Deriving τ^-_ℓ and τ^+_ℓ respectively. The following values are used for the parameters in Equations (19) and (20) :

- speed increase: $\{\underline{w}^{\min}, \underline{w}^{\max}\} = \{0.01, 0.44\}$
- speed decrease: $\{\overline{w}^{\min}, \overline{w}^{\max}\} = \{0, 0.85\}$
- for both cases: $\{\tau^{\min}, \tau^{\max}\} = \{0.01, 0.0009\}$

Interpolation techniques are used to estimate the parameter values. As we observe a stronger control on speed bounds (see Fig. 5 for illustration), we adopted Equation (16) for the remainder of this paper for computing the flight duration between two consecutive waypoints.

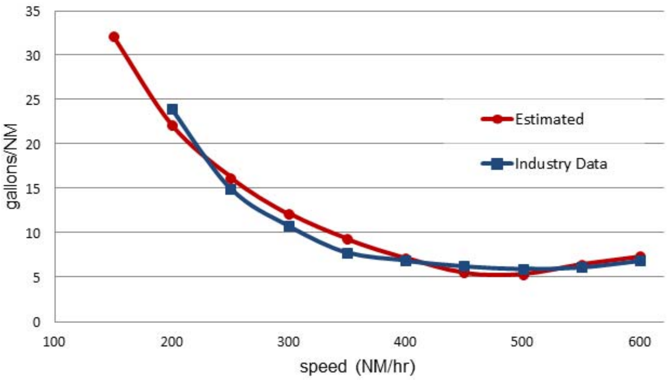


Fig. 6. Estimating industry data for fuel-consumption-rate as function of speed.

5) Fuel-Consumption Constraints: One of the major contributions of this paper is the modeling of fuel-consumption as a function of speed. Several factors including aircraft type, weather condition, flight altitude, aircraft takeoff weight and its speed impact the fuel-consumption. Except for the speed and flight altitude, none of the other factors are controllable during flight. Accordingly, we only focus on the relationship between speed and the fuel-consumption; and model the fuel-consumption as a function of flight speed.

Let FCR^f_ℓ be the amount of fuel required to fly an aircraft per nautical mile with a given speed s. Then, the cost of traveling the entire link is:

$$\mathrm{FCOST}^f_\ell = \mathrm{FCOST\_U} \times \mathrm{LENGTH}_\ell \times \mathrm{FCR}^f_\ell \tag{21}$$

where $\mathrm{FCOST\_U}$ is the unit cost of aircraft fuel and LENGTH_ℓ is the length of the given link. In Clarke *et al.* [12], a relationship between speed and fuel-consumption is established from industry data, similar to the trend illustrated in Fig. 6, for various aircraft types. Although the fuel-consumption-rate is different for each aircraft, a similar speed and fuel-consumption-rate relationship can be established for most aircraft types.

In this study, we compiled a data for the Boeing 777-200LR as a reference. Similar trends for other aircraft are illustrated in Clarke *et al.* [12]. In the model, the fuel-consumption-rate is expressed as a function of decision variable τ. As shown in Fig. 6, when plotted, τ against actual speed s, a strong correlation with the fuel-consumption-rate of the Boeing 777-200LR is observed. For $s \ge s^\star$, an inverse relationship is observed up to 600 NM/hr (maximum speed of the Boeing 777-200LR is 510 NM/hr) where $s^\star$ is the optimum speed to minimize fuel-consumption. Consequently, we scaled the τ and s relationship through scaling parameters k^1 and k^2 and obtained the following expression as the speed-dependent fuel-consumption-rate.

$$\mathrm{FCR}^f_\ell = \begin{cases} \mathrm{FCR}^{f\star}_\ell\left(k^1 x^f_\ell + k^2\left(\tau^f_\ell - \dfrac{x^f_\ell}{s^\star}\right)\right) & \text{if } \tau^f_\ell \ge \dfrac{x^f_\ell}{s^\star} \\ \mathrm{FCR}^{f\star}_\ell\left(k^1 x^f_\ell + k^2\left(\dfrac{x^f_\ell}{s^\star} - \tau^f_\ell\right)\right) & \text{otherwise.} \end{cases} \tag{22}$$

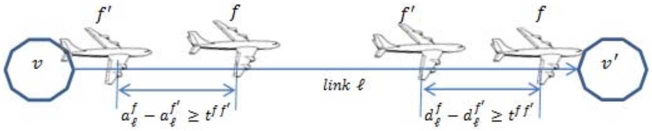


Fig. 7. Separation distance between two consecutive flights.

where $\text{FCR}_{s^\star}^f$ is the fuel-consumption-rate per unit distance traveled at the optimum speed $s^\star$. For scaling parameters $k^1 = 0.8$ and $k^2 = 1,000$ and the optimum speed $s^\star = 480$ NM/hr (estimated from the industry data provided in Clarke *et al.* [12]), the fuel-consumption and speed relationship given in Fig. 6 is obtained. It is evident from Fig. 6 that Equation (22) estimates industry data with high accuracy. Consequently, constraints (21) and (22) enable us to incorporate speed-dependent fuel-consumption cost in the objective function as given in Equation 21. It should be noted that the speed and fuel-consumption relationship is only an approximation for the steady-state conditions. During ascending and due to environmental factors (wind direction), such relationship may not be as accurate.

6) Safety and Conflict Constraints: The proposed MILP model aims at assisting ATCOs and airline companies to sustain a mid-air conflict-free ATC. Let us now introduce a set of constraints to ensure a minimum separation between aircraft, and to avoid head-on collision and intersection conflicts. For all $v \in V^-, \ell \in \omega^+(v), f, f' \in F : f < f'$

$$d_\ell^{f'} - d_\ell^f \geq t^{ff'} - M(1 - \beta_\ell^{ff'}) - M(2 - x_\ell^f - x_\ell^{f'}) \quad (23)$$

$$d_\ell^f - d_\ell^{f'} \geq t^{ff'} - M\beta_\ell^{ff'} - M(2 - x_\ell^f - x_\ell^{f'}) \quad (24)$$

$$a_\ell^{f'} - a_\ell^f \geq t^{ff'} - M(1 - \beta_\ell^{ff'}) - M(2 - x_\ell^f - x_\ell^{f'}) \quad (25)$$

$$a_\ell^f - a_\ell^{f'} \geq t^{ff'} - M\beta_\ell^{ff'} - M(2 - x_\ell^f - x_\ell^{f'}) \quad (26)$$

Inequalities (23) - (26) ensure that when two aircraft are following each other on the same link, a minimum separation time of $t^{ff'}$ is sustained. Binary decision variable $\beta_\ell^{ff'} = 1$ implies that flight f is the leader on link ℓ. The situation is illustrated in Fig. 7.

Inequalities, (27) and (28) are utilized to avoid head-on collisions. The binary decision variable $\alpha_\ell^{ff'} = 1$ implies that flight f occupies the link earlier than f' when two aircraft use the same link from opposite directions. Hence these two aircraft are separated from each other for at least $t^{ff'}$ units of time at the waypoints that defines the links.

For all $\ell \in L$, $f, f' \in F : f < f'$ where $\text{OPP}(\ell)$ is the link flow opposite to ℓ:

$$d_{\text{OPP}(\ell)}^{f'} - a_\ell^f \geq t^{ff'} - M(1 - \alpha_\ell^{ff'}) - M(2 - x_\ell^f - x_{\text{OPP}(\ell)}^{f'}) \quad (27)$$

$$a_\ell^f - d_{\text{OPP}(\ell)}^{f'} \geq t^{ff'} - M(\alpha_\ell^{ff'}) - M(2 - x_\ell^f - x_{\text{OPP}(\ell)}^{f'}) \quad (28)$$

Finally, inequalities (29) and (30) are included in the model to guarantee the sufficient separation ($t^{ff'}$) between two aircraft that are passing through the same waypoint. The binary decision variable $\theta_v^{ff'} = 1$ implies that the aircraft f passes through waypoint v before aircraft f'.

For all $v \in V^-$, $f, f' \in F : f < f'$

$$\begin{aligned} \sum_{\ell \in \omega^-(v)} d_\ell^{f'} - \sum_{\ell \in \omega^-(v)} d_\ell^f \geq t^{ff'} - M(1 - \theta_v^{ff'}) \\ - M(2 - \sum_{\ell \in \omega^-(v)} x_\ell^f - \sum_{\ell \in \omega^-(v)} x_\ell^{f'}) \end{aligned} \quad (29)$$

$$\begin{aligned} \sum_{\ell \in \omega^-(v)} d_\ell^f - \sum_{\ell \in \omega^-(v)} d_\ell^{f'} \geq t^{ff'} - M\theta_v^{ff'} \\ - M(2 - \sum_{\ell \in \omega^-(v)} x_\ell^f - \sum_{\ell \in \omega^-(v)} x_\ell^{f'}) \end{aligned} \quad (30)$$

IV. SOLUTIONS AND RESULTS

The en-route flight planning model discussed in Section III is designed to serve for both current ATC-centered (centralized) and FFC-based (decentralized) ATFM philosophies. In the centralized flight management system, the en-route flight plans for all airplanes are optimally determined at the beginning of the planning horizon. For the decentralized case, which mimics NASA's FFC ([20]), an en-route flight plan for each aircraft is determined sequentially (according to their arrival/departure sequence) given that the flight plans for all earlier flights are already determined (known). Our experiments show that, despite the fast convergence to a solution, the decentralized method suffers from two aspects: i) flights entering the airspace later in the sequence are unfairly scheduled; and ii) airspace utilization is lower. Consequently, a hybrid solution method is proposed to overcome the computational complexity of the centralized model and the quality issues with results obtained from decentralized model. Below, the experimental setup and the proposed solution strategies are discussed in detail.

A. Data Instances

In order to test the capabilities of the proposed mathematical model, two hypothetical airspaces: i) around an airport with 34 waypoints and 192 connecting links; and ii) multi-airport airspace with 50 waypoints and 170 connection links are designed. The airport example enables us to generate busy links where conflict and collision avoidance constraints can be tested extensively. Moreover, the airport example demonstrate how the proposed MILP model can assist ATCOs for sequencing aircraft arrivals and departures safely. On the other hand, the multi-airport airspace example shows how the proposed MILP model can be utilized as part of the ATFM system.

For the airport example, an aircraft enters (or exits from) the airspace from dummy waypoints (v_1^{D} and v_2^{D}). All aircraft are forced to use a single runway which is a bi-directional arc connected to the internal dummy waypoint (v_2^{D}). The external dummy waypoint (v_1^{D}) is connected to four transition waypoints for the aircraft to enter/exit the airspace. Time of entry to the airspace and the purpose of the flight (arrival or departure) are randomly generated. It is assumed that 50% of the flights are arrivals.

For the multi-airport airspace example, five waypoints are selected as airports. The departure and destination airports and the departure time of an aircraft are generated randomly. The capacity for airports and handling of the aircraft in the airport are not considered as part of this work.

Time Between Arrivals (TBA) are assumed to be following exponential distribution. Length of each link is determined based on their locations in the airspace. Links near the runway are shorter. For an aircraft approaching the airspace from outside, the entry speed is assumed to be 300 NM/hr. Minimum speed on the runway is 150 NM/hr. Between two consecutive links, the aircraft is allowed to change its speed by approximately 50% at lower speeds and up to 20% at higher speeds with a higher and lower bound, $s_\ell^f \approx [150, 550]$NM/hr. It should be noted that speed parameters may not reflect the actual flight condition. In reality, different aircraft models have different speed bounds. The cost of aircraft fuel is estimated to be \$3/gallon. Finally a pair of aircraft is separated from each other by a Separation Distance (SD) which is measured in time ($t^{ff'}$). Through various traffic conditions with a range of SD and average TBA, the impact of SD and TBA on the given objectives (average flight time in airspace, average cost and program execution times) is studied. Corresponding mathematical models were solved using IBM ILOG CPLEX Optimization Studio 12.2, using Optimization Programming Language (OPL) on a personnel computer with 64 bit operating system, 3.40 GHz Intel Core i7-2600 CPU and 16.0 GB RAM.

B. Centralized Solution Strategy

From the ATC authorities point of view, it is strongly desirable to optimize the usage of entire airspace for a given period at the beginning of the planning horizon. Hence, the flight plans for all aircraft are predetermined for the given period as: $R^f = (x_\ell^f,\ a_\ell^f,\ d_\ell^f,\ t_\ell^f,\ s_\ell^f,\ \mathrm{FCR}_\ell^f,\ \forall \ell \in L{:}x_\ell^f = 1)$. The centralized solution strategy is best suited for managing the air-traffic around airports or within individual air sectors. Despite providing the optimum space utilization, the centralized solution strategy is not practical to tackle large-scale air-traffic problems due to computational complexity. Keeping in mind that the proposed mathematical model not only handles the scheduling problem but also successfully integrates the speed-dependent fuel-consumption and collision avoidance features in one unified formulation. Hence, the computational complexity is high.

C. Decentralized Solution Strategy

In the decentralized solution strategy, we modeled and solved the MILP problem according to the principles of FFC. An aircraft departs or lands at an airport independently from the other aircraft according to its schedule. The objective is to determine the best flight plan for the approaching/departing aircraft with respect to the current traffic conditions. Hence, the problem is solved for a single aircraft given that flight plans of earlier flights ($R^f\ \forall f \in F$) are known. Despite showing very strong computational performance, the decentralized solution strategy leads to sub-optimal solutions, particularly when the airspace is heavily congested. Since the best available routes are allocated for the earlier flights, later flights are forced to take less desirable routes.

TABLE I

CENTRALIZED SOLUTION: IMPACT OF TBA AND SD ON FLIGHT TIME IN SINGLE-AIRPORT AIRSPACE AND DELAY COST

Number of Flights	TBA (Seconds)	SD (Seconds)	Execution Time (Seconds)	Average Flight Time in airspace (minutes)	Average Cost (\$)
4	30	30	0.33	6.4245	38.5
8	30	30	1.63	6.4245	38.5
12	30	30	4.44	6.4869	39.8
16	30	30	13.43	6.7113	44.2
20	30	30	413.93	7.1043	52.1
4	30	60	0.35	6.4250	38.5
8	30	60	2.04	6.4245	38.5
12	30	60	6.88	7.0740	51.5
16	30	60	273.60	8.1496	73.0
20	30	60	Out of Memory		
4	15	30	0.25	6.4250	19.3
8	15	30	1.46	6.4245	38.5
12	15	30	4.92	6.4248	38.5
16	15	30	13.24	6.4250	38.5
20	15	30	60.45	6.6994	44.0
4	15	60	0.37	6.6748	43.5
8	15	60	2.64	7.1650	53.4
12	15	60	144.85	8.4490	79.0
16	15	60	Out of Memory		
20	15	60	Out of Memory		

D. Hybrid Solution Strategy

In order to addressed the weakness of the decentralized solution strategy and the computational complexity of the centralized solution strategy, a hybrid solution strategy is introduced. In the hybrid solution strategy, en-route flight plans $R^{f'}\ \forall f' \in F'$ for the next N' flights are determined given that $R^f\ \ \forall f \in F$ for the previous N flights are already known. By controlling the size of N', both the quality of results is improved, and computational time is significantly reduced. Consequently, larger sizes of problems are solved with quality results.

E. Discussion on Solution Strategies

A large number of experiments were designed by controlling the average SD, average TBA and the number of flights in the system. The centralized method for both single-airport and multi-airport examples fails to reach an optimum solution for instances with large numbers of flights. Computation times and other statistics for the single airport case is summarized in Table I. The computation times for both single and multi-airport examples for SD = 60 seconds are provided in Fig. 8. Since less congestion is observed on links for the multi-airport case, slightly larger instances can be solved on personal computer (up to 35 aircraft on 50 waypoints airspace with 5 airports). Yet, exponentially increasing computation time suggests that, the centralized approach is not suitable for handling larger traffic conditions. Consequently, a heuristics method based on the collision avoidance constraint relaxation

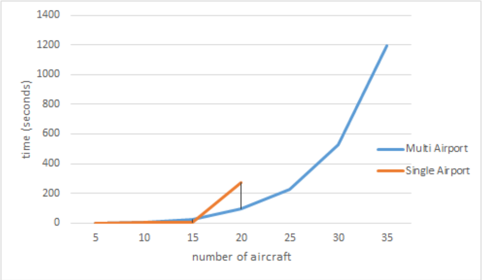


Fig. 8. Comparison of computation times for single and multi-airport cases.

has been proposed. Without the mid-air conflict avoidance constraints, the problem is reduced to a shortest path problem. Since all flights are independent from each other, the mathematical model could be solved in linear time. Following procedure is implemented.

- Solve shortest path problem $\forall f \in F$ to obtain an R^f
- Identify flights f' that violate constraints (23) - (30)
- Generate a set of flight that violets conflict constraints F^- where $F = F^- \cup F^+$
- Solve the problem $\forall f' \in F^-$ given that $R^f\ \forall f \in F^+$ are known

The proposed heuristic was able to increase the computation speed considerably (up to 48 flights on a network consists of 192 links was solved in less than 1 hour), yet the attained improvement is not sufficient to tackle general ATFM problems that concerns larger networks with multiple airports. Despite facing a major obstacle due to its computational complexity, the centralized solution strategy is a strong candidate to be adopted by ATCOs to manage the air-traffic within a single air sector or airspace near airports for short planning periods (e.g. 60 minutes or less). Furthermore, the proposed mathematical model has potential to help authorities for managing the densely populated airspace more effectively due to its capabilities of incorporating mid-air conflict avoidance and speed-dependent fuel-consumption features. The decentralized strategy on the other hand can be solved in linear time. It is an iterative approach; the MILP is solved for a single flight at a time given that the current and near future traffic conditions are known. Despite fast convergence, the decentralized strategy suffers from two aspects: i) Flight plans are determined in a sequential order based on their departure times. At the outset of the planning horizon, the airspace is empty, consequently the performance measures (cost and the flight time in airspace) for earlier flights are smaller. Hence, later flights are unfairly scheduled; ii) Since the decisions for the earlier flights are made arbitrary when the extra capacity is available at the beginning, airspace is poorly utilized. In Fig. 9, results of 8 different scenarios are illustrated. Test cases are differentiated by changing the average TBA. For all cases, a traffic size of 100 flights and SD = 0.3 minutes are used. When the airspace is congested (0.15 min. $\leq$ TBA $\leq$ 0.35 min.), the flight time in the airspace is increased and steady-state traffic conditions are not observed until the arrival of new airplanes stops (through the end of the planning horizon, total flight time is reduced due to the decreasing rate of incoming airplanes). When the capacity of the airspace is larger than the requirement (TBA $\geq$ 0.35 min.), the transition period is either short or does not exist. Existence of a steady state in 0.35 $\leq$ TBA $\leq$ 0.4 minutes arrival rate indicates the maximum capacity of the airspace for the given SD.

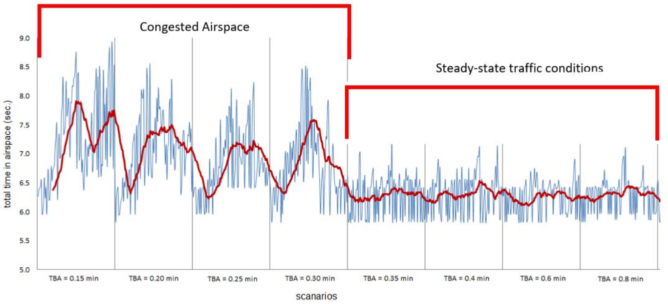


Fig. 9. Impact of TBA on flight time in airspace: X axis includes a set of experiments with different TBAs; Red line is the moving average.

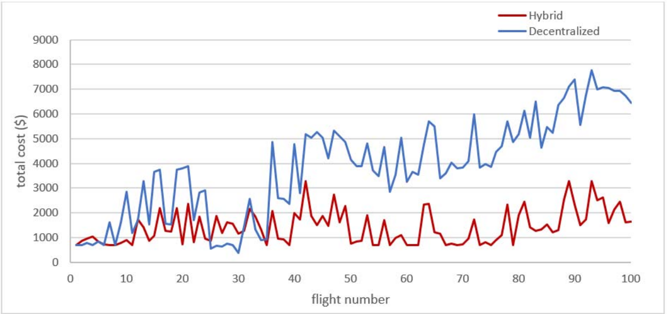


Fig. 10. Performance comparison-hybrid vs decentralized for 100 flights: TBA = 0.3 and SD = 0.31.

The hybrid solution strategy is on the other hand designed for overcoming the computational challenge of the centralized and the poor performance of the decentralized solution strategies. Since the en-route flight plan is determined for N'; new flights at each iteration, better airspace utilization and more equitable flight plans for most flights are observed. Furthermore, the computational speed is significantly improved. A comparison of Hybrid and Decentralized solution strategies for total flight cost (cost includes delay/earliness and fuel-consumption costs) for 100 flights is illustrated in Fig. 10. As evident from the figure, for TBA = 0.3 and SD = 0.31, the decentralized model fails to reach a steady state condition. Even after new flight entry to the system is stopped, the total flight costs continue increasing due to extended ground delays. On the other hand the hybrid model provides flight plans with significantly less total costs with much smaller variation. When the airspace is less densely populated (TBA = 0.75 and SD = 0.3), both decentralized and hybrid solution strategies produce compatible results for total flight costs; yet the variation among all flights under the decentralized solution strategy is significantly higher than the hybrid solution strategy (see Fig. 11 for illustration).

F. Mid-Air Conflict and Collision Avoidance

Next, we demonstrate results for the conflict and collision avoidance. Fig. 12 illustrates how aircraft sustain the desired

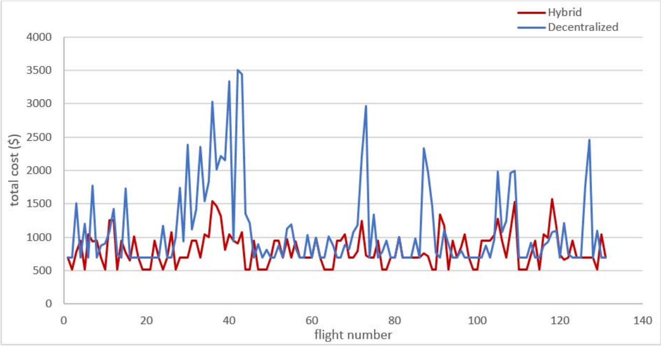
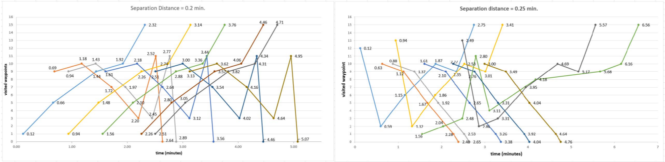


Fig. 11. Performance comparison-hybrid vs decentralized for 100 flights: TBA = 0.75 and SD = 0.3.



(a) (b)

Fig. 12. Impact of SD on flight plans: a) Flight plan for SD = 0.2 min; b) Flight plan for SD = 0.25 min.

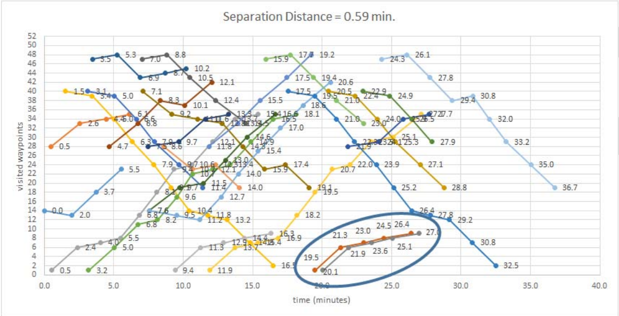


Fig. 13. Collision and conflict avoidance for the 5 airports case where SD = 0.59 minutes.

minimum separation distance during their journey. Same figure further demonstrates the impact of SD on the flight times. Finally, in Fig. 13 conflict and collision avoidance feature of the proposed MILP model is demonstrated for the multi-airport cases for 25 aircraft where SD = 0.59 minutes. In the figure, the circled area illustrates the flight route for two aircraft with the same origin and destination, following the same route while sustaining the minimum separation distance of 0.59 during their flights.

G. Airspace Capacity Optimization

In order to improve the congestion around airports, either the infrastructures need to be improved or SD should be reduced so more aircraft can be handled in the same air sector. Speijker [34] studied the possibility of reducing current SD levels in order to improve the congestion in airports. Their findings suggest that SDs can be reduced without risking the air-traffic safety. The conflict and collision avoidance features of the proposed MILP model has potentials to help aviation authorities to reduce the DC without jeopardizing the air-traffic safety. As seen in Fig. 12, when SD is smaller, aircraft reach their destinations faster, consequently airspace becomes available for the future aircraft.

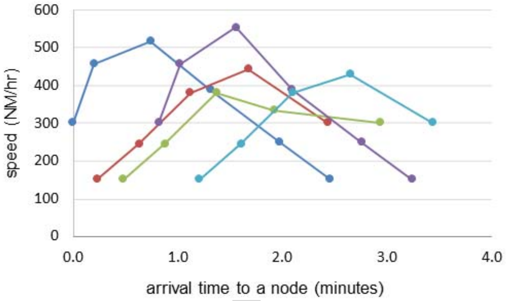


Fig. 14. Speed changes during flight.

H. Speed-Dependent Fuel-Consumption

In this work, we have approximated the fuel-consumption as a function of flight speed so that the total fuel consumed during the flight is minimized. In Fig. 14, it is shown that aircraft changes their speeds for minimizing the fuel-consumption cost (a sample of five aircraft is included in the figure).

V. CONCLUSIONS

We have presented a formulation for the ATFM problem that integrates the mid-air conflict (collision) avoidance and the speed dependent fuel-consumption issues in a unifying model. Unlike most relevant literature, the presented mathematical model avoids time-segmentation. Hence the flight times are more accurately determined. Collision avoidance and accurate computation of arrival and departure times enable decision makers to sustain the highest possible airspace utilization without jeopardizing the safety of flight which helps to overcome congestion. The provided solution strategies are practical enough whether for ATCOs to handle the entire traffic stream, or in the context of NASA's FFC, where pilots are in charge of determining their flight plans.

The presented mathematical model is a combination of scheduling and sequencing problems with conflict and collision avoidance and speed dependent fuel-consumption features. Hence the computational complexity is high. In order to address the computational challenges, a decentralized solution strategy which complies very well with the free flight philosophy and a hybrid solution strategy that provides superior results (in terms of airspace utilization and more equatable sequencing) in comparison to the decentralized strategy have been introduced.

In short, the following contributions are achieved:

- Collision avoidance is mathematically satisfied
- Airspace is more effectively used by accommodating larger number of aircraft around an airport
- Fuel-consumption cost is formulated as a function of speed
- Computational time of the model is improved by introducing decentralized and hybrid solution strategies
- Finally, the waypoint-based modeling computes traveling times much more accurately.

Due to computational complexity limitations, the centralized solution approach in this paper is not well suited for applying to all the airports and airspace in the National Airspace System (NAS) nor the entire European airspace. In fact, only small to medium size problems can be solved, with sub-optimal solutions using either the decentralized or hybrid solution strategy. Heuristic techniques such as tabu search or simulated annealing, or exact solution techniques based on column generation and lagrangian relaxation may address these computational challenges.

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Deconflicted Air-Traffic Planning With Speed-Dependent Fuel-Consumption Formulation

Ali Akgunduz, Brigitte Jaumard, *Senior Member, IEEE*, and Golbarg Moeini

***Abstract*— This paper discusses a unique formulation for the en-route flight planning problem in a constrained airspace with the objective to minimize costs incurred from earliness, lateness, and fuel-consumption; and to ensure flight safety. Mid-air conflict and collision avoidance, minimum separation distance between aircraft and speed-dependent fuel-consumption-rate are explicitly formulated. A 3D mesh network consisting of waypoints is used to provide alternative routing options for aircraft. The formulation of fuel-consumption-rate as a function of speed as part of the air-traffic planning (ATP) problem is unique in the literature. Moreover, this paper is the first attempt to model the mid-air conflict and collision avoidance as part of the ATP problem. In order to demonstrate the capabilities of the mathematical model, test instances were generated and solved by three different solution strategies. The proposed centralized solution strategy can optimally solve small size instances, similar to the air-traffic around airports to help air-traffic control authorities to manage arrival and departure sequences. Larger networks that include several airports can be solved by the proposed two sequential solution strategies (decentralized and hybrid solution strategies) to help air-traffic planning authorities to manage air-traffic safely and more economically.**

***Index Terms*— Air-traffic control, air-traffic flow management, fuel optimal control, conflict and collision avoidance, speed-dependent fuel-consumption.**

I. INTRODUCTION

DEMAND for airline services has been steadily increasing around the world. Around the world, both the seat capacity and the number of airline companies have increased significantly. While long-haul routes are still dominated by the major carriers such as United Airlines, Lufthansa or Singapore Airlines, relatively smaller airline companies have gained important market shares in the short-flight markets. These new market conditions have brought many challenges as well as benefits for the industry. Crowded airports and airspace, volatile fuel prices, increasing environmental awareness and labor costs, and unpredictable weather conditions in most parts of the world are challenging many airline companies. The transportation authorities, in particular Air-Traffic COntrollers (ATCOs) are also impacted from the current market conditions. From taxing to facilitating safe navigation in open skies, ATCOs play a crucial role in delivering on-time services and ensuring the safety of aircraft and passengers. During an aircraft's journey, its speed and route are planned by the airline, and approved and monitored by the responsible Air-Traffic Control (ATC) authorities. Pilot's discretion in the en-route flight planning process is rarely an option (pilots make real-time decisions in emergency situations). FAA anticipates an increase of 56.9% in control tower operations and over 100% in en-route (high altitude flights) traffic-control operations by 2030 [1]. It is clear that increasing air-traffic is undermining ATCOs' ability to effectively manage the given flight plans. Long working hours, stressful working conditions and continuously increasing air-traffic volume may lead to poor decision making by ATCOs, necessitating an increasing reliance on tactical collision avoidance systems embedded in airplanes to avoid mid-air collisions [23]. Furthermore, due to high air-traffic volume, safety issues must be addressed before the lower priority issues like economic and service objectives of airlines can be accommodated.

Recently, an alternative Air-Traffic Flow Management (ATFM) strategy, namely Free Flight Concept (FFC) has been proposed to reduce the workload of ATCOs and improve the air-traffic flow. FFC aims at transferring the en-route flight planning task to individual aircraft ([14], [19], and [32]). The motivation for the mathematical model introduced in this paper came from the FFC philosophy. The goal is to assist both pilots and airline companies to create safe and economic flight plans and assist ATCOs to make real-time decisions for rerouting aircraft safely and at the same time still considering the business objective of the companies. In today's air-traffic planning, an aircraft completes its journey from an origin to a destination by visiting a set of waypoints which are geographical coordinates in the sky. Hence, we propose a mathematical model that navigates aircraft through these waypoints between origin and destination airports. The waypoint based mixed-integer linear programming (MILP) model takes all meaningful airport and flight characteristics into consideration to provide a comprehensive and thorough model of the situation. The produced flight plan for each aircraft includes the sequence of waypoints to be visited, the aircraft's exact arrival and departures times at these waypoints, the average speed between consecutive

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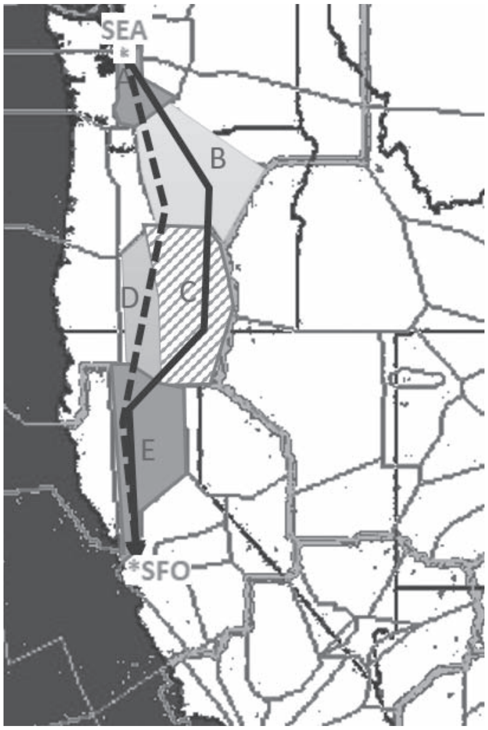


Fig. 1. Entrance and exit locations to/from the air sector impacts flight time in the air sector: Flight from Seattle to San Francisco.

waypoints and the fuel-consumption rates at that speed. While the objective is to minimize the earliness, tardiness and the fuel-consumption costs, the mid-air conflict avoidance is explicitly handled. Despite its computational complexity in comparison to alternatives available in the literature (e.g. [9]), benefits of the proposed mathematical model are crucial for the future of the air transportation. Potential benefits are:

- Increased airspace utilization: Even around the densely used airspace, the proposed waypoint based formulation provides significant flexibility for individual aircraft to perform collision-free navigation.
- Fuel-consumption: The fuel-consumption-rate as a function of speed is formulated without losing the linearity of the model. Consequently, more environmental friendly and less costly air traveling is made possible.
- Accurate traveling time: Most air sector based ATFM formulations in the literature assume that flight duration in a given air sector can be bounded by a minimum and maximum flight time. In reality on the other hand, the flight time in an air sector is strictly depends on the entrance and exit points at the air sector. As illustrated in Fig. 1, flight time in a sector would significantly vary depending on the route taken. In the proposed waypoint based formulation, we successfully addressed this problem by determining more accurate flight duration in each sector.
- Address complexity and assure accuracy: Three strategies (solution methods) are proposed. i) Centralized solution strategy where flight plans for all incoming and departing aircraft are determined at the beginning of the planning horizon; ii) Sequential (decentralized) aircraft management strategy which conforms very well with the objectives of the NASA's FFC (aims at giving the autonomy to the pilots in en-route flight planning [20]) where each airplane determines its flight plan with respect to the given current traffic conditions; iii) A Hybrid solution strategy, that combines both centralized and decentralized approaches, overcomes the scalability issue of the centralized approach and provides much better results than the decentralized approach.

The remainder of the paper is organized as follows. In Section II, a brief literature review, in Section III model, and in Section IV solution and results are introduced. Finally in Section V, the conclusions and the plans for future work are summarized.

II. LITERATURE REVIEW

In this section we discuss the relevant literature on ATC and ATFM problems. While there exists a rich literature on ATFM problems, we only cover topics that are directly related to the scope of our paper. For more general discussion on the ATFM problem, readers are referred to the works of Navazio and Romanin-Jacur [29] and de Neufville *et al.* [30]. Studies in airport and airspace congestion, mid-air conflict resolution, dynamic speed control and finally the speed-dependent fuel-consumption problems are discussed below.

A. Air-Traffic Control and Flow Management

Earlier works on ATFM focus on ATC and airport congestion problems. Airport congestion problems are further categorized as aircraft landing and take-off problems [8]. The research on ATC on the other hand mostly focuses on the technological innovations. Only a handful of operations management literature that focuses on determining and improving the size and capacity of air sectors with the objective of improving the overall performance of ATC are available ([22], [24], [28], and [38]).

More recent works in ATFM area focus on the determination of an economically sound en-route flight plan without causing congestion in the air sectors and around airports. Strategies such as ground and airborne delays/holdings are crucial for the air sector capacity management. Odoni [31] was one of the earliest to study the ground holding strategies for a single airport. Later, dynamic ground holding problem in a single airport is introduced [5], [21], [25], [27], and [37]. As the demand for air-travelling has increased, a new line of research for developing ground and airborne holding strategies for multi-airport networks has emerged [2], [15], and [36]. In their review paper, Navazio and Romanin-Jacur [29] summarized the works on multi-airport ground holding problems. It is clear that the airborne delays are more expensive than the ground holding costs. Yet, at the operational level, airborne delays are necessary to absorb the impacts of unexpected weather and air-traffic conditions. Hence, when needed, airborne delays should be handled with the least expensive ways. Consequently, the research focus has shifted on the re-routing strategies to minimize the impact of airborne delays. The rerouting concept has shifted the research focus on the ATFM problem. Consequently, the congestion problem is tackled in the entire airspace rather than at a single airport. Helm [18] introduced one of the earliest rerouting formulations. Bertsimas and Patterson [11] and later Dell'Olmo and Lulli [14] developed mathematical models for enabling rerouting to respond changing traffic and weather conditions. Other notable works on the rerouting problem are Bertsimas *et al.* [9], Leal de Matos *et al.* [13], and Ma *et al.* [26].

B. Fuel Consumption

The fuel-consumption problem in ATFM is relatively new. Most researches have focused on developing technologies to build more fuel efficient aircraft designs [7]. Regardless of the technology being used, it is known that the aircraft fuel-consumption-rate varies depending on the flight speed. Fuel-consumption is not only a cost issue. Aircraft emissions have been a major contributor to air quality, particularly around airports. However, due to the complex nature of the en-route flight planning problem, the aircraft fuel-consumption issue as part of the ATFM problem has not been studied. One of the most notable works that studies the relationship between the fuel-consumption and the speed of an aircraft is the work of Clarke *et al.* [12]. More recently, Vela et al. [35] proposed a model for conflict resolution while ensuring the optimal fuel-consumption rate. None of these works treats the fuel-consumption problem as part of the ATFM problem.

C. Mid-Air Collision Avoidance

In recent years, collision avoidance has been studied from the operations planning perspective where automated collision free path planning tools have been introduced [3], [4], [17], [33]. In their review article, Kushar and Yang [6] compare 68 conflict detection and resolutions methods using 5 different criteria: State Propagation; State Dimensions; Conflict Detection; Conflict Resolution; and Resolution Maneuvers. While some of the methods reviewed by Kushar and Yang are currently being tested and used in the industry, none of the 68 methods reviewed provides a reliable and effective solution to automate the conflict detection and resolution process in the aviation industry. Moreover, the most literature focuses on the safety aspect alone. Operational expectations such as minimization of delays and fuel-consumption are not well integrated in the conflict detection and resolution literature. The proposed mathematical model in this paper formulates the collision avoidance as part of the ATFM problem. However, the collision avoidance is only guaranteed at the waypoints. At the operational level, the proposed model must be supported by the conflict detection and resolutions methods similar to the ones discussed in Kushar and Yang [6] in order to guarantee the required separation between flights on two consecutive waypoints. Since the model described in this paper avoids conflict at the nodes, handling of conflict avoidance between waypoints is trivial.

D. Comparison With the Current Literature

The major contributions of the paper that are unique in the literature are:

- The model introduces time as a decision variable rather than periods where decisions are made only at the beginning of each period. In the current ATFM literature, state-time network is used in the formulation where $t = \{0, 1, \ldots, T\}$ is a period, and arrival and departure of an aircraft at an airspace occur in one of the predefined periods ([9] and [10]). In such formulations, the continuous notion of time is ignored.
- The formulation allows aircraft to modify its speed during flight.
- The fuel-consumption-rate as a function of speed is embedded in the ATP formulation. To the best of authors' knowledge, this is the first attempt to link fuel-consumption-rate with the aircraft speed as part of the ATP problem.
- The collision avoidance and separation distance concepts are explicitly formulated.
- The waypoint based formulation captures the real-time traffic conditions more accurately than the sector based formulations. The assumption which is common in the sector-based ATFM formulations for estimating the flight duration with a predefined bound without knowing the exact entrance and exit locations is not accurate. An aircraft's traveling time in the airspace varies significantly depending on its entrance and exit locations. Let us consider two alternative routes for an aircraft traveling from Seattle to San Francisco. As illustrated in the high altitude air route traffic control center map (Fig. 1), even though both alternative routes follow the highlighted sectors A, B, C, D, and E in the flight plans, the traveling time particularly at sectors C, D would significantly differ due to different entrance and exit locations.
- Waypoint-based en-route flight planning models are computationally more challenging in comparison to the air sector-based studies such as the one introduced in [9]. Yet, advantages such as better utilized airspace, conflict resolutions and the optimized fuel usage characteristics are sufficient to justify the additional complexity. Furthermore, decentralized and hybrid solution strategies are introduced to tackle the complexity problem.

III. PROBLEM FORMULATION

The en-route flight planning problem is formulated in a way that, all flights enter a 3D mesh network (see Fig. 2 for illustration) from one of the available entrance waypoints (υ^f_{IN}) and they travel by visiting transition waypoints (υ) through links (ℓ) to reach their destinations (υ^f_{OUT}) on time with a minimum cost and without violating safety rules. The mathematical formulation determines a flight plan (R^f) that includes a set of links ($x^f_\ell = 1$), arrival (a^f_ℓ) and departure (d^f_ℓ) times, average speed (s^f_ℓ) and the fuel-consumption-rate (FCR^f_ℓ) on these links as $R^f = (x^f_\ell, a^f_\ell, d^f_\ell, s^f_\ell, \text{FCR}^f_\ell, \forall \ell \in L : x^f_\ell = 1)$.

The details of the model are discussed in the following three subsections. First, we provide the list of parameters and decision variables. Second, the assumptions made in the modeling are summarized. Finally, the problem formulation is introduced.

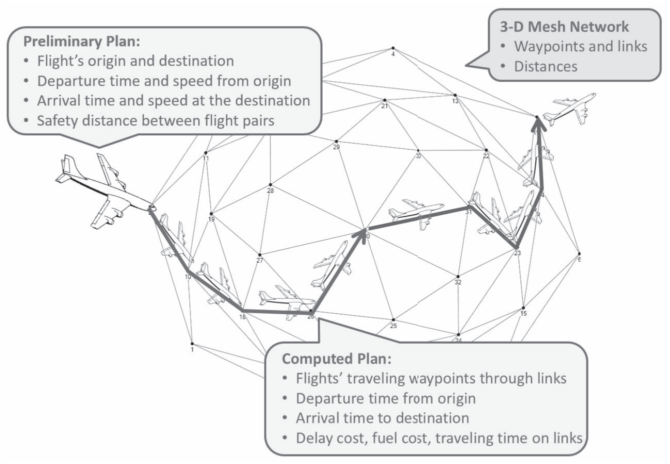


Fig. 2. Illustration of 3D mesh network used in the formulation.

A. Model Parameters and Decision Variables

1) Parameters: The definition of parameters are provided below:

F is a set of flights, indexed by f

V^+ is a set of all waypoints including entrance and exit waypoints, indexed by v

V^- is a set of transition waypoints (entrance and exit waypoints are not included)

L is a set of links, indexed by ℓ

v^f_{IN} is the entry node for flight f

v^f_{OUT} is the exit node for flight f

LENGTH_ℓ is the distance between two connected waypoints

$\omega^-_\ell(v)$ is a set of allowed incoming links for a flight leaving v through link ℓ

$\omega^+_\ell(v)$ is a set of allowed outgoing links for a flight entering v through link ℓ

The definition of ω is illustrated in Fig. 3.

t^f_{IN} and t^f_{OUT} are scheduled arrival and departure times

$t^{ff'}$ is the required separation distance (expressed in time units) for flight f' following flight f

$\tau^{\min}$ is the time to travel a unit distance with the minimum possible speed

$\tau^{\max}$ is the time to travel a unit distance with the maximum possible speed

P^f_{EARLY} and P^f_{LATE} are the penalty costs for early/late arrivals

FCOST_U is the per gallon fuel cost

In this paper, we demonstrate the feasibility of using two different speed control strategies: discrete; and continuous. Yet, only the results for continuous speed, coupled with the fuel-consumption-rate are reported in the paper. Following parameters are used for the discrete speed control policy:

S is the set of possible speeds (discretized), indexed by s

Δ is the predefined set of speed changes between two consecutive links, indexed by δ

T is the set of possible traveling times when speed is discrete, indexed by t

2) Decision Variables: Below is the definition of decision variables.

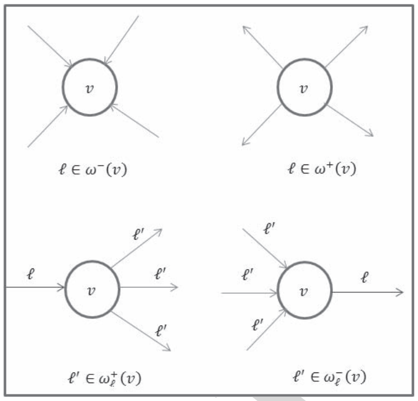


Fig. 3. Definition of parameter ω for different cases.

$$x^f_\ell = \begin{cases} 1 & \text{if flight } f \text{ travels on link } \ell \\ 0 & \text{Otherwise} \end{cases}$$

$a^f_\ell \in R^+$ is the arrival time for flight f to link $\ell \in \omega^+(v)$ from waypoint v

$d^f_\ell \in R^+$ is the departure time for flight f from link $\ell \in \omega^+(v)$ that originates from waypoint v

τ^f_ℓ is the time to travel a unit distance for flight f on link ℓ (utilized to derive actual speed)

$$\beta^{ff'}_\ell = \begin{cases} 1 & \text{flight } f' \text{ follows flight } f \text{ on link } \ell \\ 0 & \text{Otherwise} \end{cases}$$

$$\theta^{ff'}_v = \begin{cases} 1 & \text{flight } f \text{ leaves waypoint } v \text{ before} \\ & \text{flight } f' \\ 0 & \text{Otherwise} \end{cases}$$

$$\alpha^{ff'}_\ell = \begin{cases} 1 & f \text{ enters } \ell \text{ before } f' \\ & \quad \text{enters from the opposite direction} \\ 0 & \text{Otherwise} \end{cases}$$

The following decision variables are derived from the speed (τ^f_ℓ). They are independently defined to better describe the mathematical model.

FCR^f_ℓ is the fuel-consumption-rate for per unit of flight time

FCOST^f_ℓ the fuel-consumption cost in link ℓ

t^f_ℓ is the travel time for flight f on link ℓ

t^f_{EARLY} is the earliness of flight f

t^f_{LATE} is the lateness of flight f

$$x^f_{\ell s} = \begin{cases} 1 & \text{flight } f \text{ travels on } \ell \text{ with speed } s \\ 0 & \text{Otherwise} \end{cases}$$

$$y^f_{\ell \delta} = \begin{cases} 1 & \text{speed of } f \text{ is increased by } \delta \text{ on } \ell \\ 0 & \text{Otherwise} \end{cases}$$

s^f_ℓ is the speed of flight f on link ℓ

B. Assumptions

We made the following assumptions in order to realize the proposed flight planning model:

- An aircraft can visit a waypoint only once
- The speed change of an aircraft from link ℓ to the consecutive link ℓ' is bounded proportional to its current speed on link ℓ. This assumption is necessary to replicate the real flight conditions since the aircraft cannot change its speed drastically during flight

- The proposed MIP model determines only the average speed between two consecutive waypoints
- The fuel-consumption cost is determined based on the average speed
- The cost of per gallon jet fuel is assumed to be same for all types of aircraft

C. Problem Formulation

In order to solve a large scale optimization problem, it is important to obtain a strong formulation. The proposed formulation avoids non-linearity under all circumstances, yet still archives all its objectives. The described Mixed Integer Linear Programming (MILP) model considers the minimization of total cost that is incurred from delays, earliness and speed-dependent fuel-consumption. Constraints for the model are categorized in 4 groups: routing; timing; speed and fuel-consumption; and safety and conflict resolution.

1) Objective Function: Let us first define the objective function that is used in all case studies discussed later in section IV.

$$\min \sum_{f\in F} P^f_{\text{EARLY}}\, t^f_{\text{EARLY}} + P^f_{\text{LATE}}\, t^f_{\text{LATE}} + \sum_{\ell\in L} \text{FCOST}^f_\ell \tag{1}$$

The cost incurred from earliness, tardiness and fuel-consumption during flight is minimized. The relationship between speed and fuel-consumption is discussed later in the paper. Cost of delays are the collection of airport penalties, additional fuel usage and labor cost (pilots and flight attendance). In 2015, it was estimated that the cost of per minute delay for airline companies is \$65.43 [16].

2) Flight Routing Constraints: Following constraints ensure that a given aircraft travels from its origin to the destination by traveling through available waypoints.

For all $f \in F$:

$$\sum_{\ell\in\omega(v)} x^f_\ell = 1, \quad v \in v^f_{\text{IN}} \tag{2}$$

$$\sum_{\ell\in\omega(v)} x^f_\ell = 1, \quad v \in v^f_{\text{OUT}} \tag{3}$$

$$\sum_{\ell\in\omega^-(v)} x^f_\ell = \sum_{\ell'\in\omega^+(v)} x^f_{\ell'}, \quad v \in V^- \tag{4}$$

$$x^f_\ell + \sum_{\ell'\in\omega^+_\ell(v)} x^f_{\ell'} \le 1, \quad \ell \in L \tag{5}$$

$$\sum_{\ell\in\omega^-(v)} x^f_\ell \le 1, \quad v \in V \tag{6}$$

$$\sum_{\ell\in\omega^+(v)} x^f_\ell \le 1, \quad v \in V \tag{7}$$

Constraints (2) and (3) ensure all flights depart from their origins and reach their destinations. Conservation constraint (4) forces all flights entering a transition waypoint to leave the waypoint. As illustrated earlier in Fig. 3, depending on the link that an aircraft arrives at a waypoint, there are only a limited set of links available for the aircraft to leave the waypoint. The constraint (5) is utilized to limit the aircraft's departure links. Inequalities (6) and (7) limit an aircraft to visit a waypoint and a link only once (Assumption 1).

3) Timing Constraints: Next, we introduce a set of constraints to control the relationship between arrival and departure times on waypoints and links.

For all $f \in F$:

$$x^f_\ell t^f_{\text{IN}} \le a^f_\ell \tag{8}$$

$$a^f_\ell \le M x^f_\ell, \quad \ell \in L \tag{9}$$

$$d^f_\ell \le M x^f_\ell, \quad \ell \in L \tag{10}$$

$$\sum_{\ell\in\omega(v)^-} a^f_\ell = \sum_{\ell'\in\omega(v)^+} d^f_{\ell'}, v \in V \setminus \{v^f_{\text{IN}}, v^f_{\text{OUT}}\} \tag{11}$$

$$\sum_{\ell\in\omega(v^f_{\text{OUT}})^-} d^f_\ell = t^f_{\text{OUT}} + t^f_{\text{LATE}} - t^f_{\text{EARLY}} \tag{12}$$

Inequality (8) enforces aircraft to respect earliest departure times. Constraints (9) and (10) force arrival or departure times to be zero if the link is not used. In constraint (11), it is assured that the aircraft is not delayed at the intermediate waypoint. Finally, in Equation (12), exact earliness or tardiness is determined. In our case, arrival time at a waypoint is equivalent to the departure time from the connecting link (d^f_ℓ).

4) Speed Control Constraints: In the proposed mathematical model, the flight time between two consecutive waypoints is determined based on the flight speed. The distance between two consecutive waypoints (LENGTH_ℓ) is known. Therefore, the traveling time on a given link ℓ with an average speed (s^f_ℓ) is:

$$t^f_\ell = \text{LENGTH}_\ell / s^f_\ell, \quad \ell \in L, f \in F \tag{13}$$

which is a nonlinear term. In order to avoid the non-linearity, two different speed control policies can be adopted: *(i)* Discrete speed control; and *(ii)* Continuous speed control in which the speed is substituted by the *time to travel a unit distance.*

- Discrete speed control constraints

Flight time as a function of speed can easily be derived by utilizing a discrete variable. In a given link ℓ with a known link length (LENGTH_ℓ), for any speed in the speed set ($s \in S = \{s_1, \ldots, s_n\}$) there exists a corresponding flight time as $t_{\ell s} \in \text{T}_\ell$. A binary decision variable $x^f_{\ell s}$ is utilized to connect current speed with the flight duration. Consequently,

$$d^f_\ell = a^f_\ell + \sum_{s\in S(s^f_\ell)} x^f_{\ell s} t_{\ell s} \quad \ell \in L, \ f \in F \tag{14}$$

is derived. While discrete speed control is easier to model, segmentation of speed increases the computational complexity and reduces the accuracy. Consequently, a continuous speed control policy is formulated.

- Continuous speed control constraints

Let τ_ℓ be the required time to fly a unit distance with a given speed on link ℓ, where $\tau_\ell = 1 / s_\ell$. Describing speed in terms of time to travel a unit distance enables us to determine the traveling time on a link with a linear expression as:

$$t^f_\ell = \tau^f_\ell \text{LENGTH}_\ell, \quad \ell \in L, \ f \in F \tag{15}$$

Consecutively:

$$d^f_\ell = a^f_\ell + \tau^f_\ell \text{LENGTH}_\ell, \ \ell \in L, \ f \in F \tag{16}$$

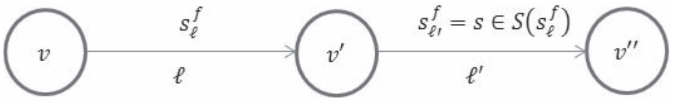


Fig. 4. Speed change from one link to the other.

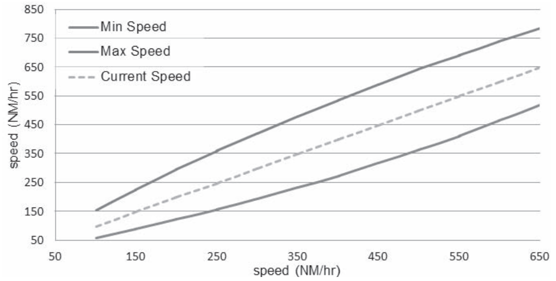


Fig. 5. Speed control between consecutive links (speed is expressed in Nautical Mile - NM per hour).

Assume that the speed on the consecutive link depends on aircraft's current speed (see Fig. 4 for illustration). Hence, a relationship similar to the Equation (17) is required to bound the speed changes from the current link to the next one.

$$(1-p_1)\tau_{\ell'}^f \leq \tau_\ell^f \leq (1+p_2)\tau_{\ell'}^f \quad \ell \in L,\ \ell' \in \omega_\ell^-(v),\ f \in F \tag{17}$$

However, constant speed change parameters (p_1 and p_2: allowable speed reduction and increase rates) may lead to significant speed changes from one link to the consecutive one. In this paper, in order to sustain a smooth transition between two consecutive links, a proportional speed change policy is adopted (speed change from one link to the next link is bounded). A numerical study revealed that, Equation (18) along with Equations (19) and (20) would imitate the desired speed-control policies as illustrated in Fig. 5. By calibrating smoothing parameters $w^{\min}$ and $w^{\max}$, a variety of speed bounds can be generated as a function of current speed τ_ℓ^f.

$$\tau_\ell^f - \tau_\ell^{f-} \leq \tau_{\ell'}^f \leq \tau_\ell^f + \tau_\ell^{f+},\ \ell \in L,\ f \in F \tag{18}$$

In Equation (18), speed increase τ_ℓ^+ and speed decrease τ_ℓ^- limits are assumed to be flight specific and determined as:

$$\tau_\ell^{f-} = \underline{w}^{\min}(\tau^{\min} - \tau_\ell^f) + \underline{w}^{\max}(\tau_\ell^f - \tau^{\max}) \tag{19}$$

$$\tau_\ell^{f+} = \overline{w}^{\min}(\tau^{\min} - \tau_\ell^f) + \overline{w}^{\max}(\tau_\ell^f - \tau^{\max}) \tag{20}$$

In Fig. 5, we plot the speed s_ℓ^f on a unit distance. It is then bounded as follows:

$$\underline{s}_\ell^f = 1/(\tau_\ell^f + \tau_\ell^{f+}) \leq s_\ell^f \leq \overline{s}_\ell^f = 1/(\tau_\ell^f - \tau_\ell^{f-})$$

Equations (19) and (20) are used to calculate Deriving τ_ℓ^- and τ_ℓ^+ respectively. The following values are used for the parameters in Equations (19) and (20) :

- speed increase: $\{\underline{w}^{\min}, \underline{w}^{\max}\} = \{0.01, 0.44\}$
- speed decrease: $\{\overline{w}^{\min}, \overline{w}^{\max}\} = \{0, 0.85\}$
- for both cases: $\{\tau^{\min}, \tau^{\max}\} = \{0.01, 0.0009\}$

Interpolation techniques are used to estimate the parameter values. As we observe a stronger control on speed bounds (see Fig. 5 for illustration), we adopted Equation (16) for the remainder of this paper for computing the flight duration between two consecutive waypoints.

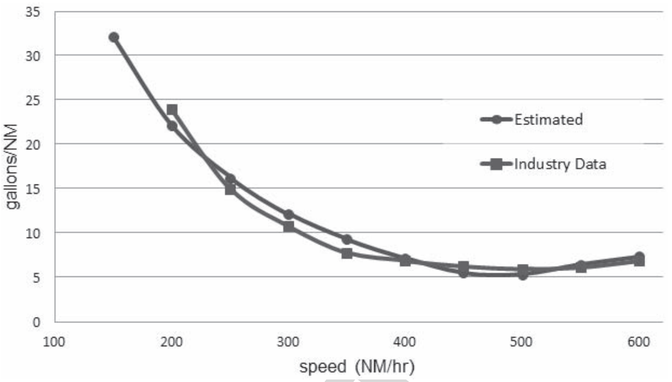


Fig. 6. Estimating industry data for fuel-consumption-rate as function of speed.

5) Fuel-Consumption Constraints: One of the major contributions of this paper is the modeling of fuel-consumption as a function of speed. Several factors including aircraft type, weather condition, flight altitude, aircraft takeoff weight and its speed impact the fuel-consumption. Except for the speed and flight altitude, none of the other factors are controllable during flight. Accordingly, we only focus on the relationship between speed and the fuel-consumption; and model the fuel-consumption as a function of flight speed.

Let FCR_ℓ^f be the amount of fuel required to fly an aircraft per nautical mile with a given speed s. Then, the cost of traveling the entire link is:

$$\text{FCOST}_\ell^f = \text{FCOST\_U} \times \text{LENGTH}_\ell \times \text{FCR}_\ell^f \tag{21}$$

where FCOST_U is the unit cost of aircraft fuel and LENGTH_ℓ is the length of the given link. In Clarke *et al.* [12], a relationship between speed and fuel-consumption is established from industry data, similar to the trend illustrated in Fig. 6, for various aircraft types. Although the fuel-consumption-rate is different for each aircraft, a similar speed and fuel-consumption-rate relationship can be established for most aircraft types.

In this study, we compiled a data for the Boeing 777-200LR as a reference. Similar trends for other aircraft are illustrated in Clarke *et al.* [12]. In the model, the fuel-consumption-rate is expressed as a function of decision variable τ. As shown in Fig. 6, when plotted, τ against actual speed s, a strong correlation with the fuel-consumption-rate of the Boeing 777-200LR is observed. For $s \geq s^\star$, an inverse relationship is observed up to 600 NM/hr (maximum speed of the Boeing 777-200LR is 510 NM/hr) where $s^\star$ is the optimum speed to minimize fuel-consumption. Consequently, we scaled the τ and s relationship through scaling parameters k^1 and k^2 and obtained the following expression as the speed-dependent fuel-consumption-rate.

$$\text{FCR}_\ell^f = \begin{cases} \text{FCR}_\ell^{f\star}\left(k^1 x_\ell^f + k^2\left(\tau_\ell^f - \dfrac{x_\ell^f}{s^\star}\right)\right) & \text{if } \tau_\ell^f \geq \dfrac{x_\ell^f}{s^\star} \\ \text{FCR}_\ell^{f\star}\left(k^1 x_\ell^f + k^2\left(\dfrac{x_\ell^f}{s^\star} - \tau_\ell^f\right)\right) & \text{otherwise.} \end{cases} \tag{22}$$

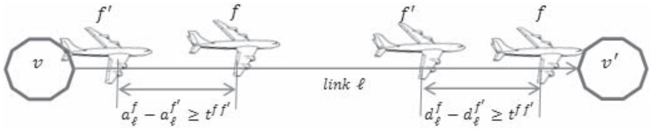


Fig. 7. Separation distance between two consecutive flights.

where $\text{FCR}^f_{s^\star}$ is the fuel-consumption-rate per unit distance traveled at the optimum speed $s^\star$. For scaling parameters $k^1 = 0.8$ and $k^2 = 1,000$ and the optimum speed $s^\star = 480$ NM/hr (estimated from the industry data provided in Clarke *et al.* [12]), the fuel-consumption and speed relationship given in Fig. 6 is obtained. It is evident from Fig. 6 that Equation (22) estimates industry data with high accuracy. Consequently, constraints (21) and (22) enable us to incorporate speed-dependent fuel-consumption cost in the objective function as given in Equation 21. It should be noted that the speed and fuel-consumption relationship is only an approximation for the steady-state conditions. During ascending and due to environmental factors (wind direction), such relationship may not be as accurate.

6) Safety and Conflict Constraints: The proposed MILP model aims at assisting ATCOs and airline companies to sustain a mid-air conflict-free ATC. Let us now introduce a set of constraints to ensure a minimum separation between aircraft, and to avoid head-on collision and intersection conflicts. For all $v \in V^-, \ell \in \omega^+(v), f, f' \in F : f < f'$

$$d^{f'}_\ell - d^f_\ell \ge t^{ff'} - M(1 - \beta^{ff'}_\ell) - M(2 - x^f_\ell - x^{f'}_\ell) \quad (23)$$

$$d^f_\ell - d^{f'}_\ell \ge t^{ff'} - M\beta^{ff'}_\ell - M(2 - x^f_\ell - x^{f'}_\ell) \quad (24)$$

$$a^{f'}_\ell - a^f_\ell \ge t^{ff'} - M(1 - \beta^{ff'}_\ell) - M(2 - x^f_\ell - x^{f'}_\ell) \quad (25)$$

$$a^f_\ell - a^{f'}_\ell \ge t^{ff'} - M\beta^{ff'}_\ell - M(2 - x^f_\ell - x^{f'}_\ell) \quad (26)$$

Inequalities (23) - (26) ensure that when two aircraft are following each other on the same link, a minimum separation time of $t^{ff'}$ is sustained. Binary decision variable $\beta^{ff'}_\ell = 1$ implies that flight f is the leader on link ℓ. The situation is illustrated in Fig. 7.

Inequalities, (27) and (28) are utilized to avoid head-on collisions. The binary decision variable $\alpha^{ff'}_\ell = 1$ implies that flight f occupies the link earlier than f' when two aircraft use the same link from opposite directions. Hence these two aircraft are separated from each other for at least $t^{ff'}$ units of time at the waypoints that defines the links.

For all $\ell \in L$, $f, f' \in F : f < f'$ where $\text{OPP}(\ell)$ is the link flow opposite to ℓ:

$$d^{f'}_{\text{OPP}(\ell)} - a^f_\ell \ge t^{ff'} - M(1 - \alpha^{ff'}_\ell) - M(2 - x^f_\ell - x^{f'}_{\text{OPP}(\ell)}) \quad (27)$$

$$a^f_\ell - d^{f'}_{\text{OPP}(\ell)} \ge t^{ff'} - M(\alpha^{ff'}_\ell) - M(2 - x^f_\ell - x^{f'}_{\text{OPP}(\ell)}) \quad (28)$$

Finally, inequalities (29) and (30) are included in the model to guarantee the sufficient separation ($t^{ff'}$) between two aircraft that are passing through the same waypoint. The binary decision variable $\theta^{ff'}_v = 1$ implies that the aircraft f passes through waypoint v before aircraft f'.

For all $v \in V^-$, $f, f' \in F : f < f'$

$$\sum_{\ell \in \omega^-(v)} d^{f'}_\ell - \sum_{\ell \in \omega^-(v)} d^f_\ell \ge t^{ff'} - M(1 - \theta^{ff'}_v) - M(2 - \sum_{\ell \in \omega^-(v)} x^f_\ell - \sum_{\ell \in \omega^-(v)} x^{f'}_\ell) \quad (29)$$

$$\sum_{\ell \in \omega^-(v)} d^f_\ell - \sum_{\ell \in \omega^-(v)} d^{f'}_\ell \ge t^{ff'} - M\theta^{ff'}_v - M(2 - \sum_{\ell \in \omega^-(v)} x^f_\ell - \sum_{\ell \in \omega^-(v)} x^{f'}_\ell) \quad (30)$$

IV. SOLUTIONS AND RESULTS

The en-route flight planning model discussed in Section III is designed to serve for both current ATC-centered (centralized) and FFC-based (decentralized) ATFM philosophies. In the centralized flight management system, the en-route flight plans for all airplanes are optimally determined at the beginning of the planning horizon. For the decentralized case, which mimics NASA's FFC ([20]), an en-route flight plan for each aircraft is determined sequentially (according to their arrival/departure sequence) given that the flight plans for all earlier flights are already determined (known). Our experiments show that, despite the fast convergence to a solution, the decentralized method suffers from two aspects: i) flights entering the airspace later in the sequence are unfairly scheduled; and ii) airspace utilization is lower. Consequently, a hybrid solution method is proposed to overcome the computational complexity of the centralized model and the quality issues with results obtained from decentralized model. Below, the experimental setup and the proposed solution strategies are discussed in detail.

A. Data Instances

In order to test the capabilities of the proposed mathematical model, two hypothetical airspaces: i) around an airport with 34 waypoints and 192 connecting links; and ii) multi-airport airspace with 50 waypoints and 170 connection links are designed. The airport example enables us to generate busy links where conflict and collision avoidance constraints can be tested extensively. Moreover, the airport example demonstrate how the proposed MILP model can assist ATCOs for sequencing aircraft arrivals and departures safely. On the other hand, the multi-airport airspace example shows how the proposed MILP model can be utilized as part of the ATFM system.

For the airport example, an aircraft enters (or exits from) the airspace from dummy waypoints (v^{D}_1 and v^{D}_2). All aircraft are forced to use a single runway which is a bi-directional arc connected to the internal dummy waypoint (v^{D}_2). The external dummy waypoint (v^{D}_1) is connected to four transition waypoints for the aircraft to enter/exit the airspace. Time of entry to the airspace and the purpose of the flight (arrival or departure) are randomly generated. It is assumed that 50% of the flights are arrivals.

For the multi-airport airspace example, five waypoints are selected as airports. The departure and destination airports and the departure time of an aircraft are generated randomly. The capacity for airports and handling of the aircraft in the airport are not considered as part of this work.

Time Between Arrivals (TBA) are assumed to be following exponential distribution. Length of each link is determined based on their locations in the airspace. Links near the runway are shorter. For an aircraft approaching the airspace from outside, the entry speed is assumed to be 300 NM/hr. Minimum speed on the runway is 150 NM/hr. Between two consecutive links, the aircraft is allowed to change its speed by approximately 50% at lower speeds and up to 20% at higher speeds with a higher and lower bound, $s_\ell^f \approx [150, 550]$NM/hr. It should be noted that speed parameters may not reflect the actual flight condition. In reality, different aircraft models have different speed bounds. The cost of aircraft fuel is estimated to be \$3/gallon. Finally a pair of aircraft is separated from each other by a Separation Distance (SD) which is measured in time ($t^{ff'}$). Through various traffic conditions with a range of SD and average TBA, the impact of SD and TBA on the given objectives (average flight time in airspace, average cost and program execution times) is studied. Corresponding mathematical models were solved using IBM ILOG CPLEX Optimization Studio 12.2, using Optimization Programming Language (OPL) on a personnel computer with 64 bit operating system, 3.40 GHz Intel Core i7-2600 CPU and 16.0 GB RAM.

B. Centralized Solution Strategy

From the ATC authorities point of view, it is strongly desirable to optimize the usage of entire airspace for a given period at the beginning of the planning horizon. Hence, the flight plans for all aircraft are predetermined for the given period as: $R^f = (x_\ell^f, a_\ell^f, d_\ell^f, t_\ell^f, s_\ell^f, \mathrm{FCR}_\ell^f, \forall \ell \in L{:}x_\ell^f = 1)$. The centralized solution strategy is best suited for managing the air-traffic around airports or within individual air sectors. Despite providing the optimum space utilization, the centralized solution strategy is not practical to tackle large-scale air-traffic problems due to computational complexity. Keeping in mind that the proposed mathematical model not only handles the scheduling problem but also successfully integrates the speed-dependent fuel-consumption and collision avoidance features in one unified formulation. Hence, the computational complexity is high.

C. Decentralized Solution Strategy

In the decentralized solution strategy, we modeled and solved the MILP problem according to the principles of FFC. An aircraft departs or lands at an airport independently from the other aircraft according to its schedule. The objective is to determine the best flight plan for the approaching/departing aircraft with respect to the current traffic conditions. Hence, the problem is solved for a single aircraft given that flight plans of earlier flights ($R^f\ \forall f \in F$) are known. Despite showing very strong computational performance, the decentralized solution strategy leads to sub-optimal solutions, particularly when the airspace is heavily congested. Since the best available routes are allocated for the earlier flights, later flights are forced to take less desirable routes.

TABLE I

CENTRALIZED SOLUTION: IMPACT OF TBA AND SD ON FLIGHT TIME IN SINGLE-AIRPORT AIRSPACE AND DELAY COST

Number of Flights	TBA (Seconds)	SD (Seconds)	Execution Time (Seconds)	Average Flight Time in airspace (minutes)	Average Cost ($)
4	30	30	0.33	6.4245	38.5
8	30	30	1.63	6.4245	38.5
12	30	30	4.44	6.4869	39.8
16	30	30	13.43	6.7113	44.2
20	30	30	413.93	7.1043	52.1
4	30	60	0.35	6.4250	38.5
8	30	60	2.04	6.4245	38.5
12	30	60	6.88	7.0740	51.5
16	30	60	273.60	8.1496	73.0
20	30	60	Out of Memory		
4	15	30	0.25	6.4250	19.3
8	15	30	1.46	6.4245	38.5
12	15	30	4.92	6.4248	38.5
16	15	30	13.24	6.4250	38.5
20	15	30	60.45	6.6994	44.0
4	15	60	0.37	6.6748	43.5
8	15	60	2.64	7.1650	53.4
12	15	60	144.85	8.4490	79.0
16	15	60	Out of Memory		
20	15	60	Out of Memory		

D. Hybrid Solution Strategy

In order to addressed the weakness of the decentralized solution strategy and the computational complexity of the centralized solution strategy, a hybrid solution strategy is introduced. In the hybrid solution strategy, en-route flight plans $R^{f'}\ \forall f' \in F'$ for the next N' flights are determined given that $R^f\ \forall f \in F$ for the previous N flights are already known. By controlling the size of N', both the quality of results is improved, and computational time is significantly reduced. Consequently, larger sizes of problems are solved with quality results.

E. Discussion on Solution Strategies

A large number of experiments were designed by controlling the average SD, average TBA and the number of flights in the system. The centralized method for both single-airport and multi-airport examples fails to reach an optimum solution for instances with large numbers of flights. Computation times and other statistics for the single airport case is summarized in Table I. The computation times for both single and multi-airport examples for SD = 60 seconds are provided in Fig. 8. Since less congestion is observed on links for the multi-airport case, slightly larger instances can be solved on personal computer (up to 35 aircraft on 50 waypoints airspace with 5 airports). Yet, exponentially increasing computation time suggests that, the centralized approach is not suitable for handling larger traffic conditions. Consequently, a heuristics method based on the collision avoidance constraint relaxation

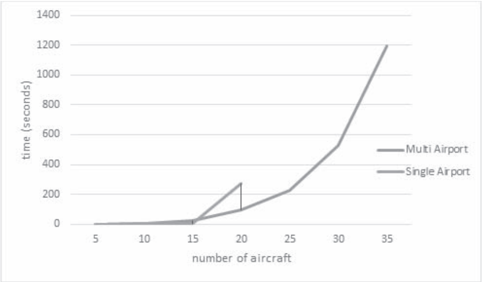


Fig. 8. Comparison of computation times for single and multi-airport cases.

has been proposed. Without the mid-air conflict avoidance constraints, the problem is reduced to a shortest path problem. Since all flights are independent from each other, the mathematical model could be solved in linear time. Following procedure is implemented.

- Solve shortest path problem $\forall f \in F$ to obtain an R^f
- Identify flights f' that violate constraints (23) - (30)
- Generate a set of flight that violets conflict constraints F^- where $F = F^- \cup F^+$
- Solve the problem $\forall f' \in F^-$ given that R^f $\forall f \in F^+$ are known

The proposed heuristic was able to increase the computation speed considerably (up to 48 flights on a network consists of 192 links was solved in less than 1 hour), yet the attained improvement is not sufficient to tackle general ATFM problems that concerns larger networks with multiple airports. Despite facing a major obstacle due to its computational complexity, the centralized solution strategy is a strong candidate to be adopted by ATCOs to manage the air-traffic within a single air sector or airspace near airports for short planning periods (e.g. 60 minutes or less). Furthermore, the proposed mathematical model has potential to help authorities for managing the densely populated airspace more effectively due to its capabilities of incorporating mid-air conflict avoidance and speed-dependent fuel-consumption features. The decentralized strategy on the other hand can be solved in linear time. It is an iterative approach; the MILP is solved for a single flight at a time given that the current and near future traffic conditions are known. Despite fast convergence, the decentralized strategy suffers from two aspects: i) Flight plans are determined in a sequential order based on their departure times. At the outset of the planning horizon, the airspace is empty, consequently the performance measures (cost and the flight time in airspace) for earlier flights are smaller. Hence, later flights are unfairly scheduled; ii) Since the decisions for the earlier flights are made arbitrary when the extra capacity is available at the beginning, airspace is poorly utilized. In Fig. 9, results of 8 different scenarios are illustrated. Test cases are differentiated by changing the average TBA. For all cases, a traffic size of 100 flights and SD = 0.3 minutes are used. When the airspace is congested (0.15 min. $\leq$ TBA $\leq$ 0.35 min.), the flight time in the airspace is increased and steady-state traffic conditions are not observed until the arrival of new airplanes stops (through the end of the planning horizon, total flight time is reduced due to the decreasing rate of incoming airplanes). When the capacity of the airspace is larger than the requirement (TBA $\geq$ 0.35 min.), the transition period is either short or does not exist. Existence of a steady state in 0.35 $\leq$ TBA $\leq$ 0.4 minutes arrival rate indicates the maximum capacity of the airspace for the given SD.

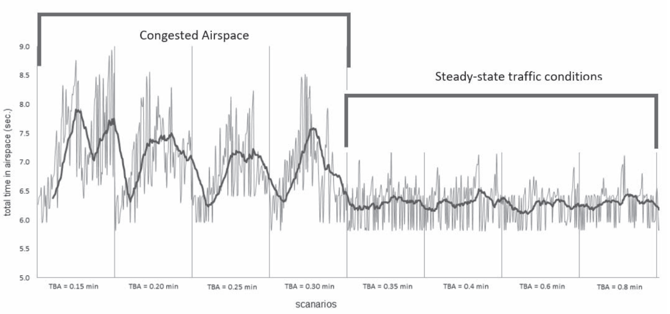


Fig. 9. Impact of TBA on flight time in airspace: X axis includes a set of experiments with different TBAs; Red line is the moving average.

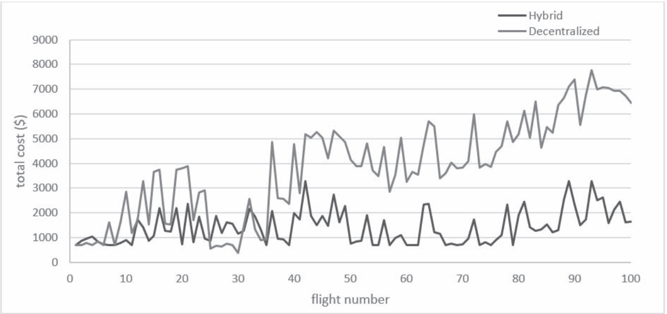


Fig. 10. Performance comparison-hybrid vs decentralized for 100 flights: TBA = 0.3 and SD = 0.31.

The hybrid solution strategy is on the other hand designed for overcoming the computational challenge of the centralized and the poor performance of the decentralized solution strategies. Since the en-route flight plan is determined for N'; new flights at each iteration, better airspace utilization and more equitable flight plans for most flights are observed. Furthermore, the computational speed is significantly improved. A comparison of Hybrid and Decentralized solution strategies for total flight cost (cost includes delay/earliness and fuel-consumption costs) for 100 flights is illustrated in Fig. 10. As evident from the figure, for TBA = 0.3 and SD = 0.31, the decentralized model fails to reach a steady state condition. Even after new flight entry to the system is stopped, the total flight costs continue increasing due to extended ground delays. On the other hand the hybrid model provides flight plans with significantly less total costs with much smaller variation. When the airspace is less densely populated (TBA = 0.75 and SD = 0.3), both decentralized and hybrid solution strategies produce compatible results for total flight costs; yet the variation among all flights under the decentralized solution strategy is significantly higher than the hybrid solution strategy (see Fig. 11 for illustration).

F. Mid-Air Conflict and Collision Avoidance

Next, we demonstrate results for the conflict and collision avoidance. Fig. 12 illustrates how aircraft sustain the desired

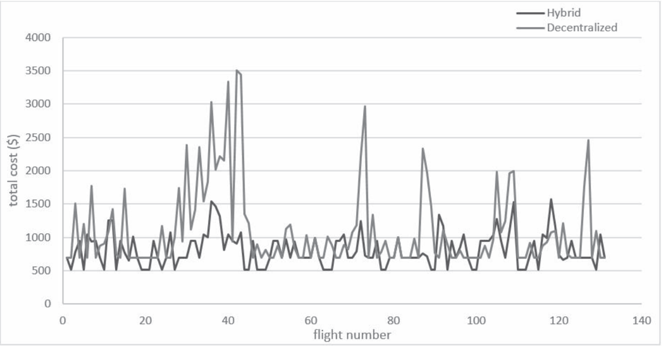
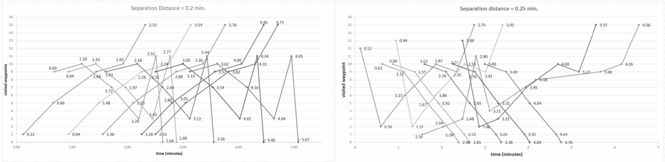


Fig. 11. Performance comparison-hybrid vs decentralized for 100 flights: TBA = 0.75 and SD = 0.3.



(a) (b)

Fig. 12. Impact of SD on flight plans: a) Flight plan for SD = 0.2 min; b) Flight plan for SD = 0.25 min.

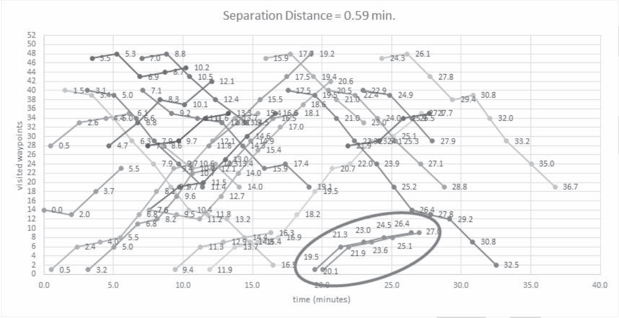


Fig. 13. Collision and conflict avoidance for the 5 airports case where SD = 0.59 minutes.

minimum separation distance during their journey. Same figure further demonstrates the impact of SD on the flight times. Finally, in Fig. 13 conflict and collision avoidance feature of the proposed MILP model is demonstrated for the multi-airport cases for 25 aircraft where SD = 0.59 minutes. In the figure, the circled area illustrates the flight route for two aircraft with the same origin and destination, following the same route while sustaining the minimum separation distance of 0.59 during their flights.

G. Airspace Capacity Optimization

In order to improve the congestion around airports, either the infrastructures need to be improved or SD should be reduced so more aircraft can be handled in the same air sector. Speijker [34] studied the possibility of reducing current SD levels in order to improve the congestion in airports. Their findings suggest that SDs can be reduced without risking the air-traffic safety. The conflict and collision avoidance features of the proposed MILP model has potentials to help aviation authorities to reduce the DC without jeopardizing the air-traffic safety. As seen in Fig. 12, when SD is smaller, aircraft reach their destinations faster, consequently airspace becomes available for the future aircraft.

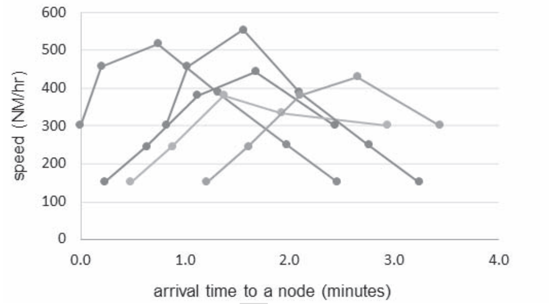


Fig. 14. Speed changes during flight.

H. Speed-Dependent Fuel-Consumption

In this work, we have approximated the fuel-consumption as a function of flight speed so that the total fuel consumed during the flight is minimized. In Fig. 14, it is shown that aircraft changes their speeds for minimizing the fuel-consumption cost (a sample of five aircraft is included in the figure).

V. CONCLUSIONS

We have presented a formulation for the ATFM problem that integrates the mid-air conflict (collision) avoidance and the speed dependent fuel-consumption issues in a unifying model. Unlike most relevant literature, the presented mathematical model avoids time-segmentation. Hence the flight times are more accurately determined. Collision avoidance and accurate computation of arrival and departure times enable decision makers to sustain the highest possible airspace utilization without jeopardizing the safety of flight which helps to overcome congestion. The provided solution strategies are practical enough whether for ATCOs to handle the entire traffic stream, or in the context of NASA's FFC, where pilots are in charge of determining their flight plans.

The presented mathematical model is a combination of scheduling and sequencing problems with conflict and collision avoidance and speed dependent fuel-consumption features. Hence the computational complexity is high. In order to address the computational challenges, a decentralized solution strategy which complies very well with the free flight philosophy and a hybrid solution strategy that provides superior results (in terms of airspace utilization and more equatable sequencing) in comparison to the decentralized strategy have been introduced.

In short, the following contributions are achieved:

- Collision avoidance is mathematically satisfied
- Airspace is more effectively used by accommodating larger number of aircraft around an airport
- Fuel-consumption cost is formulated as a function of speed
- Computational time of the model is improved by introducing decentralized and hybrid solution strategies
- Finally, the waypoint-based modeling computes traveling times much more accurately.

Due to computational complexity limitations, the centralized solution approach in this paper is not well suited for applying to all the airports and airspace in the National Airspace System (NAS) nor the entire European airspace. In fact, only small to medium size problems can be solved, with sub-optimal solutions using either the decentralized or hybrid solution strategy. Heuristic techniques such as tabu search or simulated annealing, or exact solution techniques based on column generation and lagrangian relaxation may address these computational challenges.

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