# Stable Adaptive Fuzzy Control of Nonlinear Systems Preceded by Unknown Backlash-Like Hysteresis

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***Abstract*—This paper deals with adaptive control of nonlinear dynamic systems preceded by unknown backlash-like hysteresis nonlinearities, where the hysteresis is described by a dynamic equation. By utilizing this dynamic model and by combining a fuzzy universal function approximator with adaptive control techniques, a stable adaptive fuzzy control algorithm is developed without constructing a hysteresis inverse. The stability of the closed-loop system is shown using Lyapunov arguments. The effectiveness of the proposed method is demonstrated through simulations.**

***Index Terms*—Adaptive control, fuzzy approximators, global stability, hysteresis nonlinearity, nonlinear systems, tracking control, uncertainties.**

## I. Introduction

SOME industrial actuators exhibit a hysteresis in their characteristics. In a typical piezoelectric actuator, as one of examples, hysteresis behavior is fundamentally exhibited in its response to an applied electric field. The formation of hysteresis loop of piezoelectirc actuators is a complicated procedure and physical explanation for hysteresis phenomenon from a macroscopic viewpoint was given in [2]. If the hysteresis nonlinearity in the piezoelectric actuator is not accounted for, it will result in the degradation of system performance, reducing positioning accuracy and even may lead to instability [14]. Generally speaking, hysteresis characteristics are nondifferentiable nonlinearities for which traditional control techniques are insufficient and control of a system preceded by a hysteresis is typically challenging.

The development of control techniques to mitigate effects of hystereses has been addressed in several papers [1], [3], [4], [9], [11], [15], [16]. A common feature of those schemes is that the hysteresis is patterned by the backlash hysteresis and the development of controllers relies on the construction of an inverse hysteresis. These results, especially [14] and [15], provide a theoretic framework which can serve as a base for future research. Apart from these schemes, another control approach has recently been reported in [13], where a dynamic hysteresis model is used to pattern a backlash hysteresis, which provides certain advantages.

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This paper will address the control of nonlinear systems preceded by unknown backlash hysteresis nonlinearities, for which an explicit linear parameterization of nonlinearities in the system dynamics is either unknown or impossible. The existence of such nonlinearities impose a great challenge for the controller development when the controlled system is preceded by the hysteresis. To address such a challenge, the fuzzy system will be adopted to model the plant and the controller is constructed based on this fuzzy model so that fuzzy IF–THEN rules describing the plant can be incorporated into the adaptive fuzzy controller. The fuzzy system, used to approximate the nonlinearities in the plant, together with the dynamic hysteresis model given in [13], is adjusted by adaptive laws based on a Lyapunov synthesis approach. The developed control law ensures global stability of the adaptive fuzzy system. Simulations performed on a nonlinear system illustrate and clarify the approach.

## II. Problem Statement

The controlled system consists of a nonlinear plant preceded by a backlash-like hysteresis actuator, that is, the hysteresis is present as an input of the nonlinear plant. It is a challenging task of major practical interests to develop a control scheme for both unknown system dynamics and unknown backlash-like hysteresis. The development of such a control scheme will now be pursued.

A backlash-like hysteresis nonlinearity can be denoted as an operator

$$w(t) = P[v](t) \tag{1}$$

with $v(t)$ as input $v(t)$ and $w(t)$ as output $w(t)$. The notation $[\cdot](t)$ represents the fact that the operator in $[\cdot]$ is dependent on the trajectory, $v \in C^{\circ}[0,t]$, not an instantaneous value $v(t)$. The operator $P(v(t))$ will be discussed in details in the subsequent section. The nonlinear dynamic system being preceded by the above hysteresis is described in the canonical form

$$x^{(n)}(t) + f(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = bw(t) \tag{2}$$

where $f$ is an unknown continuous nonlinear function, and control gain $b$ is unknown but constant. It is a common assumption that the sign of $b$ is known. From this point onward, without losing generality, we shall assume $b > 0$. It should be noted that more general classes of nonlinear systems can be transformed into this structure [6].

The control objective is to design a control law for $v(t)$ in (1), to force the plant state vector, $\mathbf{x} =$

$\left[x, \dot{x}, \ldots, x^{(n-1)}\right]^T$, to follow a specified desired trajectory, $\mathbf{x}_d = \left[x_d, \dot{x}_d, \ldots, x_d^{(n-1)}\right]^T$, i.e., $\mathbf{x} \to \mathbf{x}_d$ as $t \to \infty$. The following assumption regarding the desired trajectory $\mathbf{x}_d$ is required in the paper.

*Assumption 1:* The desired trajectory, $\mathbf{x}_d = \left[x_d, \dot{x}_d, \ldots, x_d^{(n-1)}\right]^T$ is continuous and available. Furthermore $\left[\mathbf{x}_d^T, x_d^{(n)}\right]^T \in \Omega_d \subset R^{n+1}$ with $\Omega_d$ a compact set.

## III. Backlash-Like Hysteresis Model and its Properties

Traditionally, a backlash hysteresis nonlinearity can be described by

$$\begin{aligned} w(t) =& P[v](t) \\ =& \begin{cases} c(v(t)-B) & \text{if } \dot{v}(t) > 0 \text{ and } w(t) = c(v(t)-B) \\ c(v(t)+B) & \text{if } \dot{v}(t) < 0 \text{ and } w(t) = c(v(t)+B) \\ w(t_-) & \text{otherwise} \end{cases} \end{aligned} \tag{3}$$

where $c > 0$ is the slope of the lines and $B > 0$ is the backlash distance. This model is itself discontinuous and may not be amenable to controller design for the nonlinear systems (2).

Instead of using the previous model, in this paper we use a continuous-time dynamic model to describe a class of backlash-like hysteresis, as given by [13]

$$\frac{dw}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv - w) + B_1\frac{dv}{dt} \tag{4}$$

where $\alpha$, $c$, and $B_1$ are constants, satisfying $c > B_1$.

*Remark:* Generally, modeling hysteresis noninearities is still a research topic and the reader may refer to [7] for a recent review.

We shall now review the solution properties of the dynamic model (4) and explain the corresponding switching mechanism, which is crucial for design of the controller. The (4) can be solved explicitly for $v$ piecewise monotone

$$\begin{aligned} w(t) =& cv(t) + d(v) \text{ with} \\ d(v) =& [w_o - cv_o]e^{-\alpha(v - v_o)\text{sgn}\dot{v}} \\ &+ e^{-\alpha v \text{sgn}\dot{v}}\int_{v_o}^{v}[B_1 - c]e^{\alpha\zeta(\text{sgn}\dot{v})}d\zeta \end{aligned} \tag{5}$$

for $\dot{v}$ constant and $w(v_o) = w_0$. Analyzing (5), we see that it is composed of a line with the slope $c$, together with a term $d(v)$. For $d(v)$, it can be easily shown that if $w(v; v_o, w_o)$ is the solution of (5) with initial values $(v_o, w_o)$, then, if $\dot{v} > 0$ ($\dot{v} < 0$) and $v \to +\infty$ $(-\infty)$, one has

$$\lim_{v\to\infty} d(v) = \lim_{v\to\infty}[w(v; v_o, w_o) - f(v)] = -\frac{c - B_1}{\alpha} \tag{6}$$

$$\left(\lim_{v\to-\infty} d(v) = \lim_{v\to-\infty}[w(v; v_o, w_o) - f(v)] = \frac{c - B_1}{\alpha}\right). \tag{7}$$

It should be noted that the aforementioned convergence is exponential at the rate of $\alpha$. Solution (5) and properties (6) and (7) show that $w(t)$ eventually satisfies the first and second conditions of (3). Furthermore, setting $\dot{v} = 0$ results in $\dot{w} = 0$ which satisfies the last condition of (3). This implies that the dynamic (4) can be used to model a class of backlash-like hystereses and is an approximation of backlash hysteresis (3).

Let us use an example for specified initial data to show the switching mechanism for the dynamic model (4) when $\dot{v}$ changes direction. We note that when $\dot{v} > 0$ on $w(0) = 0$ and $v(0) = 0$, the solution (5) gives

$$w(t) = cv(t) - \frac{c - B_1}{\alpha}(1 - e^{-\alpha v(t)}) \text{ for } v(t) \geq 0 \text{ and } \dot{v} > 0. \tag{8}$$

Let $v_s$ be a positive value of $v$ and consider now a specimen such that $v$ is increasing along the initial curve (8) until a time $t_s$ at which $v$ reaches the level $v_s$. Suppose now that from the time $t_s$, the signal $v$ is decreased. In this case, $w$ is given by

$$w(t) = cv(t) + \frac{c - B_1}{\alpha}\left[1 - (2e^{-\alpha v_s} - e^{-2\alpha v_s})e^{\alpha v(t)}\right] \quad \text{for } \dot{v} < 0. \tag{9}$$

where $v < v_s$. Equations (8) and (9) indeed show that $w$ switches exponentially from the line $cv(t) - (c - B_1)/\alpha$ to $cv(t) + (c - B_1)/\alpha$ to generate backlash-like hysteresis curves.

## IV. Lyapunov-Based Control Structure

From the solution structure (5) of the model (4) we see that the signal $w(t)$ is expressed as a linear function of input signal $v(t)$ plus a bounded term. Now it is ready for the controller development. However, before we describe the control scheme, as will be given in Section VI, it would be helpful to first develop a control law for an ideal situation. Such a control law will outline the basic structure for the controller to be developmen t. Therefore, in this section, assuming the parameter $b$ and the nonlinear function $f(\mathbf{x}(t))$ as well as the hystereis are all known, we shall propose a Lyapunov-based control structure for plants of the form described by (2), preceded by the hysteresis described in (4). The proposed controller will lead to global stability and yields desired tracking.

Using the solution expression (5), (2) becomes

$$x^{(n)}(t) + f\left(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\right) = bcv(t) + bd(v(t)) \tag{10}$$

which results in a linear relation to the input signal $v(t)$. It is very important to note that (6) or (7) imply that there exists a uniform bound $\rho$ such that

$$\|d(v)\| \leq \rho. \tag{11}$$

In presenting the Lyapunov-based control structure, the following definition is required:

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d \tag{12}$$

where $\tilde{\mathbf{x}}$ represents the tracking error vector.

A filtered tracking error is defined as

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{(n-1)}\tilde{x}(t) \text{ with } \lambda > 0. \tag{13}$$

*Remarks:* It has been shown in [10] that the equation $s(t) = 0$ defines a time-varying hyperplane in $R^n$ on which the tracking error vector $\tilde{\mathbf{x}}(t)$ decays exponentially to zero.

Define a continuous function

$$\mathrm{sat}(s) = \begin{cases} 1 - \exp\left(\frac{-s}{\gamma}\right), & \text{if } s > 0 \\ -1 + \exp\left(\frac{s}{\gamma}\right) & \text{if } s \le 0 \end{cases}. \tag{14}$$

with $\gamma$ being any small positive constant. As $\gamma \to 0$, $\mathrm{sat}(s)$ approaches a step transition from $-1$ at $s = 0^-$ to 1 at $s = 0^+$ continuously.

With this in mind, we have the following lemma to establish the existence of an ideal control, $v^*$, that leads to $\mathbf{x} \to \mathbf{x}_d$ as $t \to \infty$.

*Lemma 1:* For the plant in (2) with the hysteresis (4) at the input, all the closed-loop signals are bounded and the state vector $\mathbf{x}(t) \to \mathbf{x}_d(t)$ as $t \to \infty$ with a desired Lyapunov-based controller

$$v^*(t) = -ks(t) + v_f(\mathbf{x}(t)) + \frac{1}{bc}v_{fd}(t) - k_d\mathrm{sat}(s) \tag{15}$$

where $v_f(\mathbf{x}(t)) = (1/bc)f(\mathbf{x}(t))$, $v_{fd}(t) = x_d^{(n)}(t) - \Lambda_v^T\tilde{\mathbf{x}}(t)$ with $\Lambda_v^T = [0, \lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, \ldots, (n-1)\lambda]$, $k$ is a constant, and $k_d$ satisfies $k_d \ge \rho/c$.

*Proof:* Equation (13) can be rewritten as $s(t) = \Lambda^T\tilde{\mathbf{x}}(t)$ with $\Lambda^T = [\lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, \ldots, 1]$. The derivative of the error metric (13) can be written as

$$\begin{aligned} \dot{s}(t) =& - x_d^{(n)}(t) + \Lambda_v^T\tilde{\mathbf{x}}(t) - f(\mathbf{x}(t)) \\ &+ bcv^*(t) + bd(v^*(t)) \\ =& - v_{fd}(t) - f(\mathbf{x}(t)) + bcv^*(t) + bd(v^*(t)) \\ =& bc(-ks(t) - k_d\mathrm{sat}(s)) + bd(v^*(t)). \end{aligned} \tag{16}$$

Define a Lyapunov function candidate $V(t) = (1/2bc)s^2$. Differentiating $V$ along (16) yields

$$\begin{aligned} \dot{V}(t) =& - ks^2 - k_d s \cdot \mathrm{sat}(s) + \frac{1}{c}d(v^*(t))s \\ \le& - ks^2 \end{aligned} \tag{17}$$

where the fact that $s \cdot \mathrm{sat}(s) \ge 0$ has been used. This shows that $V$ is a Lyapunov function which leads to global boundedness of $s$. To complete the proof and establish asymptotic convergence of the tracking error, it is necessary to show that $s \to 0$ as $t \to \infty$. This is accomplished by applying Barbalat's Lemma [8] to the continuous, nonnegative function

$$\begin{aligned} V_1(t) =& V(t) - \int_0^t (\dot{V}(\tau) + ks^2(\tau))d\tau \text{ with} \\ \dot{V}_1(t) =& - ks^2(t). \end{aligned} \tag{18}$$

It can easily be shown that every term in (16) is bounded, hence, $\dot{s}$. This implies that $\dot{V}_1(t)$ is a uniformly continuous function of time. Since $V_1$ is bounded below by 0, and $\dot{V}_1(t) \le 0$ for all $t$, use of Barbalat's lemma proves that $\dot{V}_1(t) \to 0$. Therefore, from (18) it can be demonstrated that $s(t) \to 0$ as $t \to \infty$. The remark following (13) indicates that $\tilde{\mathbf{x}}(t) \to 0$ as $t \to \infty$. □□□

If the parameters $b$ and $c$ as well as the nonlinear function $f(\mathbf{x}(t))$ are unknown, the control law described above cannot directly applied. Therefore, the challenge addressed in this paper is the development of adaptive controllers to deal with unknown nonlinear function $f(\mathbf{x}(t))$ as well as the unknown parameters $b$ and $c$.

## V. Function Approximation Using Gaussian Fuzzy Membership Functions

In the case of unknown nonlinearity $f(\mathbf{x}(t))$ and constants $b$ and $c$, the desired controller $v^*(t)$ given in (15) is not available. In order to develop a stable adaptive control law, a parameterized approximator shall be used to approximate the unknown nonlinearity $v_f(\mathbf{x}(t)) = (1/bc)f(\mathbf{x}(t)$. Here, it should be emphasized that in this paper we are addressing the situation where an explicit linear parameterization of the nonlinear function $f(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t))$ is either unknown or impossible. Otherwise, there is no need to construct a parameterized approximator [18].

In the literature, several function approximators can be applied for this purpose. In this paper, we used a fuzzy system as an approximator. For the fuzzy systems, the theoretical ability to uniformly approximate continuous functions to a specified degree of accuracy has recently been demonstrated in [17] by using fuzzy IF–THEN rules, which describe the behavior of an unknown plant.

The control design presented in this paper employs a fuzzy system to approximate the function $v_f(\cdot)$ over a compact region of the state space. Such a fuzzy system is composed by *singleton fuzzifier, product inference,* and *Gaussian membership function* and is of the form

$$h(\mathbf{x}(t)) = \sum_{j=1}^{N} \omega_j(t)g_j\left(\sigma_j(t)\|\mathbf{x}(t) - \xi_j(t)\|\right) \tag{19}$$

where $h : U \subset R^n \to R$, $\omega_j(t)$ is the connection weight; $g_j\left(\sigma_j(t)\|\mathbf{x}(t) - \xi_j(t)\|\right) = \prod_{i=1}^{n} \mu_{A_j^i}(x_i(t))$ and $\mu_{A_j^i}(x_i(t))$ is the *Gaussian membership function,* defined by

$$\mu_{A_j^i}(x_i(t)) = \exp\left(-(\sigma_j^i(t)(x_i(t) - \xi_j^i(t)))^2\right) \tag{20}$$

where $\sigma_j^i(t)$ and $\xi_j^i(t)$ are real-valued parameters, $\sigma_j(t) = [\sigma_j^1(t), \sigma_j^2(t), \ldots, \sigma_j^n(t)]^T$ and $\xi_j(t) = [\xi_j^1(t), \xi_j^2(t), \ldots, \xi_j^n(t)]^T$. Notice that contrary on the traditional notation, in this paper, we use $1/\sigma_j^i(t)$ to represent the variance just for the convenience of later development.

Then, the fuzzy system (19) can be rewritten as

$$h(\mathbf{x}(t)) = W^T(t) \cdot G\left(\mathbf{x}(t), \xi(t), \sigma(t)\right) \tag{21}$$

where $W(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_N(t)]^T$, $G(\mathbf{x}(t), \xi(t), \sigma(t)) = [g_1(\sigma_1(t)\|\mathbf{x}(t) - \xi_1(t)\|), g_2(\sigma_2(t)\|\mathbf{x}(t) - \xi_2(t)\|), \ldots, g_N(\sigma_N(t)\|\mathbf{x}(t) - \xi_N(t)\|)]^T$, $\xi(t) = [\xi_1(t), \xi_2(t), \ldots, \xi_N(t)]^T$, $\sigma(t) = [\sigma_1(t), \sigma_2(t), \ldots, \sigma_N(t)]^T$.

The ability of (21) to uniformly approximate smooth functions over compact set is well documented in the literature (see [17] and [18]). In particular, it has been shown that given a smooth function $v_f(\mathbf{x}(t)) : U \mapsto R$, where $U \in R^n$ is a compact set and $\varepsilon_h > 0$, there exists a Gaussuian function vector $G\left(\mathbf{x}, \xi^*, \sigma^*\right)$ and a weight vector $W^*$ such that $\sup_{\mathbf{x} \in U} \left|v_f(\mathbf{x}(t)) - W^{*T}G\left(\mathbf{x}, \xi^*, \sigma^*\right)\right| < \varepsilon_h$.

To construct $v_f^* = W^{*T} G(\mathbf{x}, \xi^*, \sigma^*)$, the values of the parameter vectors $W^*$, $\sigma^*$, and $\xi^*$ are required. Unfortunately, they are usually unavailable. Normally, the unknown parameter vectors $W^*$, $\sigma^*$, and $\xi^*$ are replaced by their estimates $\hat{W}$, $\hat{\sigma}$, and $\hat{\xi}$. Then, the estimation function $\hat{v}_f = \hat{W}^T G\left(\mathbf{x}, \hat{\xi}, \hat{\sigma}\right)$ is used instead of $v_f^*$ to approximate the unknown function $v_f$. Using the estimation function $\hat{v}_f$ of $v_f^*$, the approximation error between $v_f(\mathbf{x}(t))$ and $\hat{v}_f(\mathbf{x}(t))$ can be established as follows.

*Lemma 2:* Define the estimation errors of the parameter vectors as

$$\tilde{W}(t) = W^* - \hat{W}; \quad \tilde{\xi}(t) = \xi^* - \hat{\xi}; \quad \tilde{\sigma}(t) = \sigma^* - \hat{\sigma}. \quad (22)$$

The estimation error function $\tilde{v}f(t) = v_f(\mathbf{x}(t)) - \hat{v}_f(\mathbf{x}(t))$ is

$$\begin{aligned}\tilde{v}_f(t) = {} & \tilde{W}^T(t) \cdot (\hat{G}(t) - G'_\xi \hat{\xi}(t) - G'_\sigma \hat{\sigma}(t)) \\ & + \hat{W}^T(t) \cdot (G'_\xi \tilde{\xi}(t) + G'_\sigma \tilde{\sigma}(t)) + d_f(t) \quad (23)\end{aligned}$$

where $G'_\xi \in R^{N \times (Nn)}$ and $G'_\sigma \in R^{N \times (Nn)}$ are derivatives of $G(\mathbf{x}(t), \xi^*, \sigma^*)$ with respect to $\xi^*$ and $\sigma^*$ at $(\hat{\xi}(t), \hat{\sigma}(t))$, respectively, $d_f(t)$ is a residual term. Moreover, $d_f(t)$ satisfies

$$|d_f(t)| < \theta_f^{*T} \cdot Y_f(t) \quad (24)$$

and $\theta_f^* \in R^{4 \times 1}$ is an unknown constant vector, being composed of optimal weight matrices and some bounded constants; and $Y_f(t) = \left[1, \|\hat{W}(t)\|, \|\hat{\xi}(t)\|, \|\hat{\sigma}(t)\|\right]^T$ is a known function vector.

*Proof:* See Appendix A

*Remarks:*

1) The role of Lemma 2 is that through the first Taylor's expansion of $v_f^*(\mathbf{x}(t))$ near $(\hat{\xi}(t), \hat{\sigma}(t))$, the function approximation error $\tilde{v}_f(t)$ in (23) has been expressed in a linearly parameterizable form with respect to $\tilde{\xi}$ and $\tilde{\sigma}$, which makes the updates of $(\hat{\xi}(t), \hat{\sigma}(t))$ possible. Moreover, the residual term is bounded by a linear expression with a known function vector as in (24). Thus, adaptive control techniques can be applied to deal with this residual term. We should mention that the similar technique has also been used in the reference [5], where the approximator was constructed by the current neural networks and the hidden neurons are sigmoid functions.
2) It should be noted that no explicit expressions for $\xi^*$, $\sigma^*$, $W^*$, $\theta_f^*$ are required since these values can be learned by using the adaptive algorithm developed in the following section.

## VI. Adaptive Controller Design

We are now ready to develop an adaptive control law to achieve the control objective for the plant described by (2), preceded by the hysteresis described in (4) with unknown nonlinear function $f(\mathbf{x}(t))$ and $b$ as well as $c$. Before proposing an adaptive control law, the following assumptions regarding the plant and hysteresis are required.

A1) There exist known constants $b_{\min}$ and $b_{\max}$ such that the control gain $b$ in (2) satisfies $b \in [b_{\min}, b_{\max}]$.

A2) There exist known constants $c_{\min}$ and $c_{\max}$ such that the slope $c$ in (3) satisfies $c \in [c_{\min}, c_{\max}]$.

A3) The bound $\rho$ for the relation $\|d(v)\| \le \rho$ is known.

*Remarks:* Assumption A1) is common for the nonlinear system described by (2) [10]. Assumption A2) assumes the slope range of a backlash hysteresis nonlinearity, which is reasonable. Assumption A3) requires knowledge in regards to the upper bound of the hysteresis loop, which is quite reasonable and practical.

In presenting the developed robust adaptive control law, the following definition is required:

$$\tilde{\phi} = \phi - \hat{\phi} \quad (25)$$

where $\hat{\phi}$ is an estimate of $\phi$, which is defined as $\phi \triangleq (bc)^{-1}$.

Given the plant in (2), hysteresis model (4) in Section III, and the function approximation (23) in Section V, subject to the assumption previously described, the following control and adaptation laws are presented:

$$\begin{aligned}v(t) = {} & -ks(t) + \hat{\phi} v_{fd}(t) + \hat{W}^T G\left(\mathbf{x}, \hat{\xi}, \hat{\sigma}\right) \\ & -(\hat{\theta}_f^T Y_f + k^*)\mathrm{sat}(s) \quad (26)\\ u_{fd}(t) = {} & x_d^{(n)}(t) - \Lambda_v^T \tilde{\mathbf{x}}(t) \quad (27)\\ \dot{\hat{W}} = {} & -\Gamma_1 s(t)(\hat{G} - G'_\xi \hat{\xi} - G'_\sigma \hat{\sigma}) \quad (28)\\ \dot{\hat{\xi}} = {} & -\Gamma_2 s(t)(\hat{W}^T G'_\xi)^T \quad (29)\\ \dot{\hat{\sigma}} = {} & -\Gamma_3 s(t)(\hat{W}^T G'_\sigma)^T \quad (30)\\ \dot{\hat{\theta}}_f = {} & \Gamma_4 |s(t)| Y_f \quad (31)\\ \dot{\hat{\phi}} = {} & \mathrm{Proj}(\hat{\phi}, -\eta v_{fd} s) \quad (32)\end{aligned}$$

where $\Lambda_v^T = \left[0, \lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, \ldots, (n-1)\lambda\right]$; $k^*$ is a control gain, satisfying $k^* \ge \rho / c_{\min}$, whereby, $\rho$ is defined in (11). In addition, the parameters $\eta$ and $\Gamma_i$ $(i = 1, \ldots, 4)$ are positive constants determining the rates of adaptations, and $\mathrm{Proj}(\cdot, \cdot)$ is a projection operator, which is formulated as follows:

$$\mathrm{Proj}(\hat{\phi}, -\eta v_{fd} s) = \begin{cases} 0 & \text{if } \hat{\phi}_i = \phi_{\max} \text{ and } \eta v_{fd} s < 0 \\ -\eta v_{fd} s & \text{if } [\phi_{\min} < \hat{\phi} < \phi_{\max}] \\ & \text{or } [\hat{\phi} = \phi_{\max} \text{ and } \eta v_{fd} s \ge 0] \\ & \text{or } [\hat{\phi} = \phi_{\min} \text{ and } \eta v_{fd} s \le 0] \\ 0 & \text{if } \hat{\phi} = \phi_{\min} \text{ and } \eta v_{fd} s > 0 \end{cases}. \quad (33)$$

*Remarks:*

1) In the aforementioned control law, a projection operator has been introduced. It can be easily proved that the projection operator for $\hat{\phi}$ has the following properties: i) $\hat{\phi}(t) \in \Omega_\phi$ if $\hat{\phi}(0) \in \Omega_\phi$; ii) $\|\mathrm{Proj}(p, y)\| \le \|y\|$; and iii) $(p^* - p)^T \Lambda \mathrm{Proj}(p, y) \ge (p^* - p)^T \Lambda y$, where $\Lambda$ is a positive–definite symmetric matrix.
2) The projection operator requires knowledge of the parameters $\phi_{\min}$ and $\phi_{\max}$. These represent the upper and lower bounds of $\phi$, respectively. Assumptions A1) and A2) are fundamental to this end. However, it should be noted that these parameters are only used to specify the range of parameter changes for the projection operator. With regards to this paper, such a range is not restricted as long as the

estimated parameter is bounded (required for the stability proof); hence, one can always choose suitable $\phi_{\min}$ and $\phi_{\max}$, although such a choice may be conservative.

The stability of the closed-loop system described by (2), (4) and (26)–(33) is established in the following theorem.

*Theorem 1:* For the plant in (2) with the hysteresis (4) at the input subject to assumptions A1)–A3), the robust adaptive controller specified by (26)–(33) ensures that if $\hat{\phi}(t_0) \in \Omega_\phi$, all the closed-loop signals are bounded and the state vector $\mathbf{x}(t)$ converges to $\mathbf{x}_d$ as $t \to \infty$.

*Proof:* Using (10), the time derivative of the filtered error (13) can be written as

$$\dot{s}(t) = -v_{fd}(t) - f(\mathbf{x}(t)) + bcv(t) + bd(v). \quad (34)$$

Using the control law (26)–(33), the previous equation can be rewritten as

$$\dot{s} = -v_{fd} - f(\mathbf{x}) + bc\Big[-ks + \hat{\phi}v_{fd} + \hat{W}^T G\left(X,\hat{\xi},\hat{\sigma}\right) -(\hat{\theta}_f^T Y_f + k^*)\mathrm{sat}(s)\Big] + bd(v). \quad (35)$$

To establish global boundedness, we define a Lyapunov function candidate

$$V(t) = \frac{1}{2}\left[\frac{1}{bc}s^2 + \frac{1}{\eta}(\phi-\hat{\phi})^2 + \tilde{W}^T\Gamma_1^{-1}\tilde{W} + \tilde{\xi}^T\Gamma_2^{-1}\tilde{\xi} +\tilde{\sigma}^T\Gamma_3^{-1}\tilde{\sigma} + \tilde{\theta}_f^T\Gamma_4^{-1}\tilde{\theta}_f\right]. \quad (36)$$

The derivative of $\dot{V}$ along (35) leads to

$$\begin{aligned}\dot{V}(t) =& \frac{1}{bc}s\dot{s} - \frac{1}{\eta}(\phi-\hat{\phi})\dot{\hat{\phi}} - \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}} - \tilde{\xi}^T\Gamma_2^{-1}\dot{\hat{\xi}}\\ &- \tilde{\sigma}^T\Gamma_3^{-1}\dot{\hat{\sigma}} - \tilde{\theta}_f^T\Gamma_4^{-1}\dot{\hat{\theta}}_f\\ =& -ks^2 + s\left[\hat{\phi}v_{fd}(t) + \hat{W}^T G\left(X,\hat{\xi},\hat{\sigma}\right)\right.\\ &\left.-(\hat{\theta}_f^T Y_f + k^*)\mathrm{sat}(s)\right] + \frac{1}{bc}s[-v_{fd} - f(\mathbf{x}) + bd(v)]\\ &- \frac{1}{\eta}(\phi-\hat{\phi})\dot{\hat{\phi}} - \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}} - \tilde{\xi}^T\Gamma_2^{-1}\dot{\hat{\xi}}\\ &- \tilde{\sigma}^T\Gamma_3^{-1}\dot{\hat{\sigma}} - \tilde{\theta}_f^T\Gamma_4^{-1}\dot{\hat{\theta}}_f\\ =& -ks^2 + s\left[\hat{\phi}v_{fd}(t) + \hat{W}^T G\left(X,\hat{\xi},\hat{\sigma}\right)\right.\\ &\left.-(\hat{\theta}_f^T Y_f + k^*)\mathrm{sat}(s)\right] + s\left[-\phi v_{fd} - v_f(\mathbf{x}) + \frac{d}{c}\right]\\ &- \frac{1}{\eta}(\phi-\hat{\phi})\dot{\hat{\phi}} - \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}} - \tilde{\xi}^T\Gamma_2^{-1}\dot{\hat{\xi}}\\ &- \tilde{\sigma}^T\Gamma_3^{-1}\dot{\hat{\sigma}} - \tilde{\theta}_f^T\Gamma_4^{-1}\dot{\hat{\theta}}_f. \end{aligned} \quad (37)$$

Using Lemma 2, one has

$$\begin{aligned} v_f(\mathbf{x}) &- \hat{W}^T G\left(X,\hat{\xi},\hat{\sigma}\right)\\ =& \frac{1}{bc}f(\mathbf{x}) - \hat{W}^T G\left(X,\hat{\xi},\hat{\sigma}\right)\\ =& \tilde{W}^T(t)\cdot(\hat{G}(t) - G'_\xi\hat{\xi}(t) - G'_\sigma\hat{\sigma}(t))\\ &+ \hat{W}^T(t)\cdot(G'_\xi\tilde{\xi}(t) + G'_\sigma\tilde{\sigma}(t)) + d_f(t). \end{aligned} \quad (38)$$

Equation (37) can then be expressed as

$$\begin{aligned}\dot{V}(t) =& -ks^2 + s[\hat{\phi}v_{fd}(t) - (\hat{\theta}_f^T Y_f + k^*)\mathrm{sat}(s)]\\ &- s[\tilde{W}^T(t)\cdot(\hat{G}(t) - G'_\xi\hat{\xi}(t) - G'_\sigma\hat{\sigma}(t))\\ &+ \hat{W}^T(t)\cdot(G'_\xi\tilde{\xi}(t) + G'_\sigma\tilde{\sigma}(t)) + d_f(t)]\\ &+ s\left[-\phi v_{fd} + \frac{d}{c}\right] - \frac{1}{\eta}(\phi-\hat{\phi})\dot{\hat{\phi}} - \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}}\\ &- \tilde{\xi}^T\Gamma_2^{-1}\dot{\hat{\xi}} - \tilde{\sigma}^T\Gamma_3^{-1}\dot{\hat{\sigma}} - \tilde{\theta}_f^T\Gamma_4^{-1}\dot{\hat{\theta}}_f. \end{aligned} \quad (39)$$

Since $|d_f| \le {\theta_f^*}^T Y_f$, this becomes

$$\begin{aligned}\dot{V}(t) \le& -ks^2 + s[\hat{\phi}v_{fd}(t) - (\hat{\theta}_f^T Y_f + k^*)\mathrm{sat}(s)]\\ &- s\tilde{W}^T(t)(\hat{G}(t) - G'_\xi\hat{\xi}(t) - G'_\sigma\hat{\sigma}(t))\\ &- s\hat{W}^T(t)(G'_\xi\tilde{\xi}(t) + G'_\sigma\tilde{\sigma}(t)) + |s|{\theta_f^*}^T Y_f\\ &+ s\left[-\phi v_{fd} + \frac{d}{c}\right] - \frac{1}{\eta}(\phi-\hat{\phi})\dot{\hat{\phi}} - \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}}\\ &- \tilde{\xi}^T\Gamma_2^{-1}\dot{\hat{\xi}} - \tilde{\sigma}^T\Gamma_3^{-1}\dot{\hat{\sigma}} - \tilde{\theta}_f^T\Gamma_4^{-1}\dot{\hat{\theta}}_f. \end{aligned} \quad (40)$$

By using adaptive law (32) and the property $-(1/\eta)(\phi - \hat{\phi})\mathrm{Proj}(\hat{\phi}, -\eta v_{fd}s) \le (\phi-\hat{\phi})v_{fd}s$, one obtains

$$\begin{aligned}\dot{V}(t) \le& ks^2 + s[\hat{\phi}v_{fd}(t) - (\hat{\theta}_f^T Y_f + k^*)\mathrm{sat}(s)]\\ &- s\tilde{W}^T(t)\cdot(\hat{G}(t) - G'_\xi\hat{\xi}(t) - G'_\sigma\hat{\sigma}(t))\\ &- s\hat{W}^T(t)(G'_\xi\tilde{\xi}(t) + G'_\sigma\tilde{\sigma}(t)) + |s|{\theta_f^*}^T Y_f\\ &- s\phi v_{fd} + |s|\frac{\rho}{c_{\min}} + (\phi-\hat{\phi})v_{fd}s - \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}}\\ &- \tilde{\xi}^T\Gamma_2^{-1}\dot{\hat{\xi}} - \tilde{\sigma}^T\Gamma_3^{-1}\dot{\hat{\sigma}} - \tilde{\theta}_f^T\Gamma_4^{-1}\dot{\hat{\theta}}_f\\ \le& -ks^2 - |s|\hat{\theta}_f^T Y_f + \left(-k^* + \frac{\rho}{c_{\min}}\right)|s|\\ &- s\tilde{W}^T(t)\cdot(\hat{G}(t) - G'_\xi\hat{\xi}(t) - G'_\sigma\hat{\sigma}(t))\\ &- s\hat{W}^T(t)(G'_\xi\tilde{\xi}(t) + G'_\sigma\tilde{\sigma}(t)) + |s|{\theta_f^*}^T Y_f\\ &- \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}} - \tilde{\xi}^T\Gamma_2^{-1}\dot{\hat{\xi}} - \tilde{\sigma}^T\Gamma_3^{-1}\dot{\hat{\sigma}} - \tilde{\theta}_f^T\Gamma_4^{-1}\dot{\hat{\theta}}_f\\ \le& -ks^2. \end{aligned} \quad (41)$$

Equation (41) implies that $V$ is a Lyapunov function which leads to global boundedness of $s$ and $(\hat{\phi}-\phi)$, as well as $\hat{W}, \hat{\xi}, \hat{\sigma}, \hat{\theta}_f$. It is easily shown that if $\tilde{\mathbf{x}}(0)$ is bounded, then $\tilde{\mathbf{x}}(t)$ is also bounded for all $t$, and since $\mathbf{x}_d(t)$ is bounded by design, $\mathbf{x}(t)$ must also be bounded. To complete the proof and establish asymptotic convergence of the tracking error, it is necessary to show that $s \to 0$ as $t \to \infty$. This is accomplished by applying Barbalat's Lemma [8] to the continuous, nonnegative function

$$\begin{aligned} V_1(t) =& V(t) - \int_0^t(\dot{V}(\tau) + ks^2(\tau))d\tau \text{ with}\\ \dot{V}_1(t) =& -ks^2(t). \end{aligned} \quad (42)$$

It can easily be shown that every term in (35) is bounded, hence, $\dot{s}$ is bounded This implies that $\dot{V}_1(t)$ is a uniformly continuous function of time. Since $V_1$ is bounded below by 0, and $\dot{V}_1(t) \le 0$ for all $t$, use of Barbalat's lemma proves that $\dot{V}_1(t) \to 0$. Therefore, from (42) it can be demonstrated that $s(t) \to 0$ as

$t \to \infty$. The remark following (13) indicates that $\tilde{\mathbf{x}}(t)$ will converge to $\tilde{\mathbf{x}}_d(t)$. □□□

*Remark:* It is important to note that the backlash-like hysteresis model described by (4) can be extended for the general hysteresis nonlinearities. However, the goal of this paper is to show the controller design strategy using a dynamic hysteresis model in a simple setting that reveals its essential features. This is the motivation for simply using backlash-like hysteresis model.

## VII. Simulation Studies

In this section, we illustrate the aforementioned methodology on a simple nonlinear system described as

$$\dot{x} = a\frac{1-e^{-x(t)}}{1+e^{-x(t)}} + bw(t) \tag{43}$$

where $w(t)$ represents an output of hysteresis. The actual parameter values are $b = 1$ and $a = 1$. Without control, i.e., $w(t) = 0$, the system (43) is unstable, because $\dot{x} = (1-e^{-x(t)})/(1+e^{-x(t)}) > 0$ for $x > 0$, and $\dot{x} = (1-e^{-x(t)})/(1+e^{-x(t)}) < 0$ for $x < 0$. The objective is to control the system state $x$ to follow a desired trajectory $x_d$, which will be specified later.

The backlash-like hysteresis is described by

$$\frac{dw}{dt} = \alpha\left|\frac{dv}{dt}\right|[cv - w] + \frac{dv}{dt}B_1 \tag{44}$$

with parameters $\alpha = 1$, $c = 3.1635$, and $B_1 = 0.345$. Using input signal $v(t) = \mathrm{ksin}(2.3t)$ with $k = 2.5$, 3.5, 4.5, 5.5, 6.5, the responses of this dynamic equation with the initial condition $w(0) = 0$ are shown in Fig. 1. We should mention that when using a variety of values for both initial values $w(0)$ and frequencies, simulation studies show hysteresis shapes similar to those in Fig. 1. This confirms again that the dynamic model (44) can be used to describe the backlash-like hysteresis. It also shows that the required shape of backlash hysteresis is dependent solely on the selection of a suitable parameter set $\{\alpha, c, B_1\}$.

To construct the fuzzy approximator approximator, the following linguistic descriptions are adopted:

$$R_f^k : \text{IF } x \text{ is near } k, \text{ THEN is near } B_k$$

where near $k$, $k = -3, -2, -1, 0, 1, 2, 3$, is a fuzzy set with membership functions $\mu_K(x) = \exp(-\sigma(x-k)^2)$. $B_k$ are obtained by evaluating $(1/bc)f(x)$ at points $x = -3, -2, -1, 0, 1, 2, 3$. The values of $B_k$ are not required here since the exact $W^*$, $\xi^*$, and $\sigma^*$ are not required in the control law. However, the knowledge of $B_k$ will be helpful in the choices of initial $\hat{W}(0)$, $\hat{\xi}(0)$, $\hat{\sigma}(0)$, and $\hat{\theta}_f(0)$ to speed up the adaptation process. In this example, these initial values are chosen as $\hat{W}(0) = (1/3.16)[-0.8, -0.6, -0.4, 0, 0.4, 0.6, 0.8]^T$, $\hat{\xi}(0) = [-3, -1, -1, 0, 1, 2, 3]^T$, $\hat{\sigma}(0) = [2, 2, 2, 2, 2, 2, 2]^T$, and $\hat{\theta}_f(0) = [0.1, 0.01, 0.01, 0.01]$.

In the simulations, the robust adaptive control law (26)–(33) was used, taking $k_d = 20$. Since the backlash distance is around 2.5, we can choose the upper bound $\rho$ in (11) as $\rho = 4$ and we also choose $c_{\min} = 3$, which results in $k^* = 4/3$. In the adaptation laws, we choose $\Gamma_i|_{i=1,2,3,4} = \{0.2, 0.2, 0.3, 0.01\}$ and $\eta = 0.5$ and the initial parameter $\phi = 0.8/3$. The initial state is chosen as $x(0) = 1.05$ and sample time is 0.005. In the simulation the initial value, $v(0)$, is required, which is selected as $v(0) = 0$.

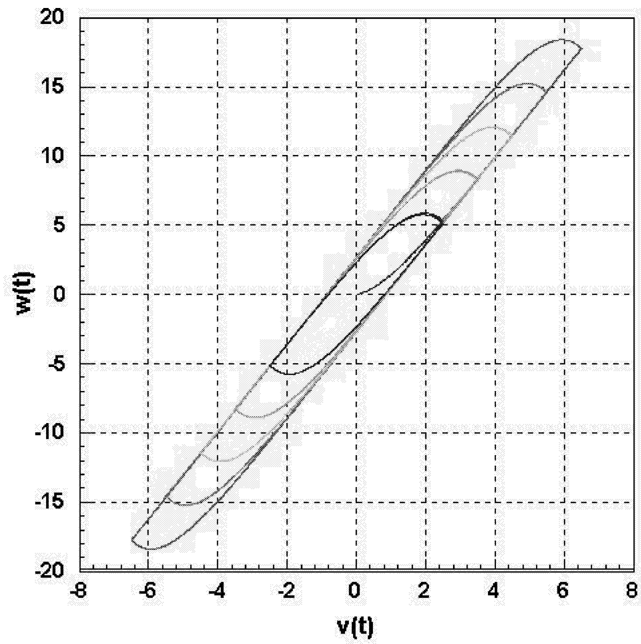


Fig. 1. Hysteresis curves given by (4) or (44) with $\alpha = 1$, $c = 3.1635$, and $B_1 = 0.345$ for $v(t) = \mathrm{kitsin}(2.3t)$ with $k = 2.5$, 3.5, 4.5, 5.5, 6.5.

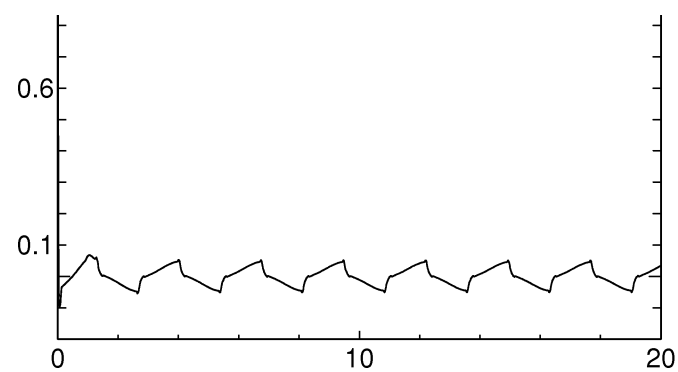


Fig. 2. Tracking error of the state with backlash hysteresis.

Choosing the desired trajectory $x_d(t) = 12.5\sin(2.3t)$, simulation results are shown in Figs. 2 and 3. Fig. 2 shows the tracking error for the desired trajectory and Fig. 3 shows the input control signal $v(t)$. We see from Fig. 2 that the proposed robust controller clearly demonstrates excellent tracking performance. We should mention that it is desirable to compare the control performance with and without considering the effects of hysteresis. Unfortunately, this comparison is not possible in this case as the control law (26)–(33) is designed for the entire cascade system.

## VIII. Conclusion

In this paper, a stable fuzzy adaptive control architecture is proposed for a class of continuous-time nonlinear dynamic systems preceded by an unknown backlash-like hysteresis, where

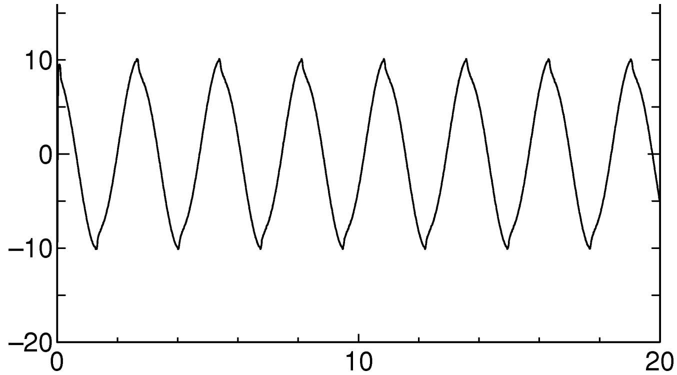


Fig. 3. Control signal $v(t)$ acting as the input of backlash hysteresis.

the backlash-like hysteresis is modeled by a dynamic equation. By showing the properties of the hysteresis model and by combining a fuzzy universal function approximator with adaptive control techniques, an adaptive control scheme is developed without constructing the hysteresis inverse. The proposed adaptive control law ensures global stability of the adaptive system and achieves the desired tracking. Simulations performed on a nonlinear system illustrate and clarify the approach.

## APPENDIX
## PROOF OF LEMMA 2

### A. Proof

Denoting $\hat{G} = G\left(\mathbf{x}, \hat{\xi}, \hat{\sigma}\right)$ and noticing $v_f(\mathbf{x}(t)) = v_f^*(\mathbf{x}) + \varepsilon_v$ with $|\varepsilon_v| \le \varepsilon_h$, one has

$$\begin{aligned}\tilde{v}f(t) =& v_f(\mathbf{x}(t)) - \hat{v}_f(\mathbf{x}(t)) \\ =& v_f^*(\mathbf{x}) - \hat{W}^T\hat{G} + \varepsilon_v \\ =& W^{*T}G^* - W^{*T}\hat{G} + W^{*T}\hat{G} - \hat{W}^T\hat{G} + \varepsilon_v \\ =& W^{*T}\tilde{G} + \tilde{W}^T\hat{G} + \varepsilon_v \\ =& W^{*T}\tilde{G} - \hat{W}^T\tilde{G} + \hat{W}^T\tilde{G} + \tilde{W}^T\hat{G} + \varepsilon_v \\ =& \tilde{W}^T\tilde{G} + \hat{W}^T\tilde{G} + \tilde{W}^T\hat{G} + \varepsilon_v \end{aligned} \tag{45}$$

where $\tilde{G} = G^* - \hat{G} = G(\mathbf{x}, \xi^*, \sigma^*) - G\left(\mathbf{x}, \hat{\xi}, \hat{\sigma}\right)$. In order to deal with $\tilde{G}$, the Taylor's series expansion of $G^*$ is taken about $\xi^* = \hat{\xi}$ and $\sigma^* = \hat{\sigma}$. This produces

$$\begin{aligned}G(\mathbf{x}, \xi^*, \sigma^*) = G\left(\mathbf{x}, \hat{\xi}, \hat{\sigma}\right) + G'_\xi \cdot (\xi^* - \hat{\xi}) \\ + G'_\sigma \cdot (\sigma^* - \hat{\sigma}) + o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})\end{aligned} \tag{46}$$

where $o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})$ denotes the sum of high-order arguments in a Taylor's serious expansion, and $G'_\xi$ and $G'_\sigma$ are derivatives of $G(\mathbf{x}, \xi^*, \sigma^*)$ with respect to $\xi^*$ and $\sigma^*$ at $(\hat{\xi}, \hat{\sigma})$. They are expressed as

$$\begin{aligned}G'_\xi =& \frac{\partial G(\mathbf{x}, \xi^*, \sigma^*)}{\partial \xi^*}\Big|_{\xi^* = \hat{\xi},\ \sigma^* = \hat{\sigma}} \\ G'_\sigma =& \frac{\partial G(\mathbf{x}, \xi^*, \sigma^*)}{\partial \sigma^*}\Big|_{\xi^* = \hat{\xi},\ \sigma^* = \hat{\sigma}}.\end{aligned} \tag{47}$$

Equation (46) can then be written as

$$\tilde{G} = G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma} + o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma}). \tag{48}$$

Using (48), $\tilde{v}f(t)$ in (45) can be expressed as

$$\begin{aligned}\tilde{v}f(t) =& \tilde{W}^T(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma} + o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})) \\ &+ \hat{W}^T(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma} + o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})) + \tilde{W}^T\hat{G} + \varepsilon_v \\ =& \tilde{W}^T(\hat{G} - G'_\xi\hat{\xi} - G'_\sigma\hat{\sigma}) + \hat{W}^T(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma}) + d_f\end{aligned} \tag{49}$$

where

$$d_f = \tilde{W}^T(G'_\xi\xi^* + G'_\sigma\sigma^*) + W^{*T}o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})) + \varepsilon_v.$$

Now, let us examine $d_f$. First, using (48), the high-order term $o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})$ is bounded by

$$\begin{aligned}\|o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})\| =& \|\tilde{G} - G'_\xi\tilde{\xi} - G'_\sigma\tilde{\sigma}\| \\ \le& \|\tilde{G}\| + \|G'_\xi\tilde{\xi}\| + \|G'_\sigma\tilde{\sigma}\| \\ \le& c_1 + c_2\|\tilde{\xi}\| + c_3\|\tilde{\sigma}\|\end{aligned} \tag{50}$$

where $c_1$, $c_2$, and $c_3$ are some bounded constants due to the fact that Gaussian function and its derivative are always bounded by constants (the proof is omitted here to save space). Second, it is obvious that there should exist constants $\bar{W}$, $\bar{\xi}$, and $\bar{\sigma}$ satisfying $\|W^*\| \le \bar{W}$, $\|\xi^*\| \le \bar{\xi}$, and $\|\sigma^*\| \le \bar{\sigma}$. Finally, based on the facts:

$$\begin{aligned}\|\tilde{W}\| \le& \|W^*\| + \|\hat{W}\| \le \|\bar{W}\| + \|\hat{W}\| \\ \|\tilde{\xi}\| \le& \|\xi^*\| + \|\hat{\xi}\| \le \|\bar{\xi}\| + \|\hat{\xi}\| \\ \|\tilde{\sigma}\| \le& \|\sigma^*\| + \|\hat{\sigma}\| \le \|\bar{\sigma}\| + \|\hat{\sigma}\|.\end{aligned}$$

The term $d_f$ can be bounded as

$$\begin{aligned}|d_f| =& \|\tilde{W}^T(G'_\xi\xi^* + G'_\sigma\sigma^*) + W^{*T}o(\mathbf{x}, \tilde{\xi}, \tilde{\sigma})) + \varepsilon_v\| \\ \le& \|\tilde{W}\|\|G'_\xi\|\|\xi^*\| + \|\tilde{W}\|\|G'_\sigma\|\|\sigma^*\| \\ &+ \|W^*\|(c_1 + c_2\|\tilde{\xi}\| + c_3\|\tilde{\sigma}\|) + \varepsilon_h \\ \le& (\bar{W} + \|\hat{W}\|)c_2\bar{\xi} + (\bar{W} + \|\hat{W}\|)c_3\bar{\sigma} + \bar{W}c_1 \\ &+ \bar{W}c_2(\bar{\xi} + \|\hat{\xi}\|) + \bar{W}c_3(\bar{\sigma} + \|\hat{\sigma}\|) + \varepsilon_h \\ =& 2c_2\bar{W}\bar{\xi} + 2c_3\bar{W}\bar{\sigma} + c_1\bar{W} + \varepsilon_h + (c_2\bar{\xi} + c_3\bar{\sigma})\|\hat{W}\| \\ &+ c_2\bar{W}\|\hat{\xi}\| + c_3\bar{W}\|\hat{\sigma}\| \\ =& [\theta_{f1}^*, \theta_{f2}^*, \theta_{f3}^*, \theta_{f4}^*] \cdot [1, \|\hat{W}\|, \|\hat{\xi}\|, \|\hat{\sigma}\|]^T \\ =& \theta_f^{*T}Y_f\end{aligned} \tag{51}$$

where $\theta_{f1}^* = 2c_2\bar{W}\bar{\xi} + 2c_3\bar{W}\bar{\sigma} + c_1\bar{W} + \varepsilon_h$, $\theta_{f2}^* = c_2\bar{\xi} + c_3\bar{\sigma}$, $\theta_{f3}^* = c_2\bar{W}$, and $\theta_{f4}^* = c_3\bar{W}$. □□□

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