Real-Time Implementation of Regressor-Based Sliding Mode Control Algorithm for Robotic Manipulators

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***Abstract*—A regressor-based variable structure control scheme has been developed for the trajectory control of robot manipulators in the presence of disturbances, parameter variations, and unmodeled dynamics. The method is based on the regressor structure given by Slotine and Li, [3], without parameter adaptation. This avoids the requirement of persistency of excitation, and the convergence of the overall transient is exponential. The method is robust against a class of state-dependent uncertainties, which may result, for example, from unmodeled dynamics. The problem of chattering is solved by the smoothing control law. It is shown that the closed-loop system is globally ultimately bounded with respect to a set around the origin, which can be made arbitrarily small. To illustrate the feasibility of this controller, it was implemented using a Motorola M68000 microprocessor on a two-link revolute joint manipulator subjected to a variable payload. Experimental results confirm the validity of accurate tracking capability and the robust performance.**

I. Introduction

The development of a modern industrial manipulator calls for robustness with regard to variable payloads, torque disturbances, parameter variations, and unmodeled dynamics. Adaptive control of robot manipulators, as an approach to the solution of the trajectory tracking problem in the presence of uncertainties, has attracted intense research interest, and a full review is given in [1]. Adaptive controllers [2]–[5] are based on the linear parameterization approach, resulting in better performance. However, in these controllers the convergence of the parameter estimation is based on the condition of persistent excitation, and even with persistency of excitation, the transients may not be uniform, and the convergence of the tracking errors to zero may be very slow [6].

Variable structure control, as an alternative for the robust approach [7], has been applied to the trajectory control of robot manipulators [8]–[21], and is receiving increasing attention. It differs from adaptive control in that the bounds of uncertainties must be known and no learning mechanism is used. It also has the advantage of a prescribed transient response in the sliding mode.

The early works on sliding mode control for the trajectory tracking of manipulators [8]–[11] are based on the bounds of the unknown parameters and uncertainties. The discontinuous control laws, which make each sliding surface attractive in order to guarantee the asymptotic stability of their intersection, are constructed respectively. Corresponding controller gains are defined by a set of fairly complicated algebraic inequalities. A sliding mode controller which avoids the inversion of the estimate of the inertia matrix were introduced in [14]. More recent works in this field [13], [15], [18], [20], which take into consideration the important properties of the robot dynamics, result in control laws that ensure the stability of the intersection of the surfaces without necessarily stabilizing each individual one. Other controllers were also developed by using a special Lyapunov function [12] or linearizable methods [17], [21].

Inspired by the regressor structure presented by Slotine and Li [3], [4], a regressor-based algorithm, based on variable structure control, is proposed in this paper. This algorithm uses Slotine's controller structure; however, there is no parameter adaptation. This provides advantages that make it particularly suitable for multiple manipulator control. First it does not need persistency of excitation; second, the convergence of the overall transient is exponential; and, finally, it is robust against uncertainties in the model. Compared with the sliding mode methods found in the literature, the main advantage of the algorithm is that the explicit requirements for guaranteeing stability are easily known, and the switching gain can explicitly be determined in terms of parameter variations rather than in matrix bounds. The proposed algorithm is also implemented on a two-revolute joint manipulator power by PWM transistor converter-fed dc servomotors.

The arrangement of this article is as follows: in Section II the robot dynamics and structure properties are reviewed. The adaptive control scheme given by Slotine and Li is briefly reviewed in Section III. Section IV presents the new control algorithm, and robustness, with respect to unmodeled dynamics, is analyzed. A smoothed control law

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is also suggested in this section to overcome the problem of chattering. Finally, the real-time experiments and results, in the presence of such uncertainty as handling a varying payload, are discussed in Section V.

II. Manipulator Dynamic Model

A manipulator is defined as an open kinematic chain of rigid links. Each degree-of-freedom of the manipulator is powered by independent torques. Using the Lagrangian formulation, the equations of motion of an n-degree-of-freedom manipulator can be written as

$$D(\boldsymbol{q})\ddot{\boldsymbol{q}} + B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) = \boldsymbol{u} \tag{1}$$

where $\boldsymbol{q} \in R^n$ is the generalized coordinate (joint positions); $D(\boldsymbol{q}) \in R^{n \times n}$ is the symmetric, bounded, positive definite inertia matrix; vector $B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in R^n$ presents the centripetal and Coriolis torques; $G(\boldsymbol{q}) \in R^n$ is the vector of gravitational torques, which is a bounded C^1 function; and $\boldsymbol{u} \in R^n$ is the vector of applied joint torques. The robot model (1) is characterized by the following structural properties, which are of central importance to the stability analysis.

Property 1: There exists a vector $\alpha \in R^m$ with components depending on manipulator parameters (masses, moments of inertia, etc.), such that

$$D(\boldsymbol{q})\dot{\boldsymbol{v}} + B(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{v} + G(\boldsymbol{q}) = \Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{v}, \dot{\boldsymbol{v}})\alpha \tag{2}$$

where $\Phi \in R^{n \times m}$ is called the regressor [2], [3], and $\boldsymbol{v}(t) \in R^n$ is a vector of smooth functions.

This property means that the dynamic equation can be linearized with respect to a specially selected set of manipulator parameters, thus constituting the basis of the *linear parameterization* approach [2]–[5].

Property 2: Using a proper definition of matrix $B(\boldsymbol{q}, \dot{\boldsymbol{q}})$, both $D(\boldsymbol{q})$ and $B(\boldsymbol{q}, \dot{\boldsymbol{q}})$ in (1) satisfy

$$x^T(\dot{D} - 2B)\boldsymbol{x} = 0, \quad \forall\, \boldsymbol{x} \in R^n$$

with $\boldsymbol{x}^T$ the transposition of $\boldsymbol{x}$. That is, $(\dot{D} - 2B)$ is a skew-symmetric matrix [3]. In particular, the elements of $B(\boldsymbol{q}, \dot{\boldsymbol{q}})$ may be defined as [1], [3]

$$B_{ij} = \frac{1}{2}\left[\dot{\boldsymbol{q}}^T \frac{\partial D_{ij}}{\partial \boldsymbol{q}} + \sum_{k=1}^{n}\left(\frac{\partial D_{ik}}{\partial q_j} - \frac{\partial D_{jk}}{\partial q_i}\right)\dot{q}_k\right]. \tag{3}$$

Property 2 is simply a statement that the so-called *fictitious forces*, defined by $B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$, do not work on the system [1].

Remark: The structure properties of the robot dynamics presented above have been used to design a sliding mode controller for the trajectory tracking problem. By using Property 2, a simple sliding mode controller was presented [13]. Other classes of sliding mode controllers [15], [18], [20] were proposed with the help of Properties 1 and 2.

Throughout this paper, the norm of vector x is defined as

$$\|x\| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$$

and that of matrix A is defined as the corresponding induced norm

$$\|A\| = \left(\max_{\text{eigenvalue}} A^T A\right)^{1/2}.$$

The singular value of matrix A is defined as $\gamma(A) = (\text{eigenvalue}(A^TA))^{1/2}$. $\gamma_{\min}(A)$ denotes the smallest singular value. The relation $x^TAx \geq \gamma_{\min}(A)\|x\|^2$, for $A = A^T > 0$ concerning $\gamma(A)$ is useful in deriving the control algorithm.

III. Brief Overview of Adaptive Controller

In this section, a brief overview of the adaptive controller proposed by Slotine and Li [3], [4] is given. The considered sliding mode controller design problem is as follows: For any given desired trajectory $\boldsymbol{q}_d \in R^n$, $\dot{\boldsymbol{q}}_d \in R^n$, and $\ddot{\boldsymbol{q}}_d \in R^n$, with some or all of the manipulator parameters unknown, derive a controller for the actuator torques, and an estimation law for the unknown parameters, such that the manipulator joint position $\boldsymbol{q}(t)$ precisely tracks $\boldsymbol{q}_d(t)$ after an initial adaptive process.

Let $\alpha = [\alpha_1 \cdots \alpha_m]^T$ be a constant m-dimensional vector containing the unknown elements in the suitably selected set of equivalent robot dynamic parameters. Let $\hat{\alpha}$ be its estimate, and let $\hat{D}$, $\hat{B}$, and $\hat{G}$ be the matrices obtained from the matrices D, B, and G by substituting the estimated $\hat{\alpha}$ for actual α. Then the linear parameterizability of the dynamics (Property 1) enables the following to be derived

$$\tilde{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}_r + \tilde{B}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_r + \tilde{G}(\boldsymbol{q}) = \Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r)\tilde{\alpha} \tag{4}$$

where $\Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r) \in R^{n \times m}$ is the regressor matrix [2], [3] independent of the dynamic parameters, $\tilde{\alpha} = \hat{\alpha} - \alpha$ is the parameter estimation error, and $\dot{\boldsymbol{q}}_r$ is defined as

$$\dot{\boldsymbol{q}}_r = \dot{\boldsymbol{q}}_d - \Lambda\tilde{\boldsymbol{q}} \tag{5}$$

where Λ is a positive definite matrix whose eigenvalues are strictly in the right-half complex plane, and $\tilde{\boldsymbol{q}}(t) = \boldsymbol{q}(t) - \boldsymbol{q}_d(t)$ denotes the position tracking error. Vector $\dot{\boldsymbol{q}}_r$ is called "reference velocity," and is introduced to guarantee the convergence of the position tracking.

The following choice of adaptive controller and adaptation law were suggested:

$$\boldsymbol{u} = \Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r)\hat{\alpha} - K_d\boldsymbol{s} \tag{6}$$

$$\dot{\hat{\alpha}} = -\Gamma\Phi^T(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r)\boldsymbol{s} \tag{7}$$

where Γ is a constant positive definite matrix, K_d is a uniformly positive definite matrix, and the vector $\boldsymbol{s}$, which can be thought of as a measure of tracking accuracy, is defined as

$$\boldsymbol{s} = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_r = \dot{\tilde{\boldsymbol{q}}} + \Lambda\tilde{\boldsymbol{q}}. \tag{8}$$

The above control and adaptation laws guarantee the global convergence of the positional and velocity tracking errors, as long as the desired trajectories q, $\dot{q}_d$, and $\ddot{q}_d$ are bounded. It was proved that s converged asymptotically to zero with the following Lyapunov function candidate

$$V(t) = \tfrac{1}{2}[s^T D s + \tilde{\alpha}^T \Gamma^{-1} \tilde{\alpha}]. \tag{9}$$

Then, from definition (8), the convergence of s to zero in turn guarantees that $\tilde{q}$ and $\dot{\tilde{q}}$ also converge to zero. Intuitively, this corresponds to the fact that the output of a stable linear filter, whose input converges to zero, must also converge to zero. Therefore, both global stability of the system, and convergence of the tracking error, are guaranteed by the above adaptive controller.

This adaptive control result represents a turning point in the literature of adaptive robot controls. The significant contribution is the initial establishment of the global tracking convergence requiring no acceleration measurements. However, since the adaptation law (7) is typically a gradient law, as remarked in [4], the guaranteed convergence of the tracking errors to zero does not imply the convergence of the estimated parameters to the exact values. It is shown in [23] that the estimated parameters asymptotically converge to the true parameters if the matrix $\Phi(q_d, \dot{q}_d, \ddot{q}_d)$ is persistently exciting and uniformly continuous. Persistent excitation means the existence of positive constants δ, α_1, and α_2 such that for all $t_1 \geq 0$

$$\alpha_1 I \leq \int_{t_1}^{t_1+\delta} \Phi_d^T \Phi_d \, dt \leq \alpha_2 I$$

An important point, as suggested in [6], is that even with persistency of excitation, the quality of the adaptation transient, i.e., while $\hat{\alpha}$ is away from α, is not uniform, and convergence of s or $\tilde{\alpha} = \hat{\alpha} - \alpha$ may be very slow [6]. It can only guarantee zero steady errors. At present, the transient analysis of adaptive systems is still in its infancy and few significant results are available. It is also noted that pure integral action may lead to well-known robustness problems under nonideal conditions [25], [26].

IV. Sliding Mode Controller

A. Controller Design

To avoid the difficulty related to the parameter estimation and further improve performance of the above adaptive controller, a variable structure control approach, combined with the controller structure described in the previous section, is now used to construct a new class of robust controllers.

Consider the plant defined in (1): the controller, given in (6), is modified with estimated parameters $\hat{\alpha}$ replaced by $\psi = [\psi_1 \cdots \psi_m]^T$, i.e.,

$$u = \Phi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\psi - K_d s \tag{10}$$

where ψ_i are switching functions designed according to the variable structure theory [9], [10] as explained below.

Taking $s = 0$, where s is defined in (8), as a sliding surface, then by combining (10) with (1) and using the fact that $s = \dot{q} - \dot{q}_r$ the sliding mode equation becomes

$$D\dot{s} = \Phi\psi + \Phi\alpha - Bs - K_d s \tag{11}$$

where the following definition has been used, which follows (4)

$$D(q)\ddot{q}_r + B(q, \dot{q})\dot{q}_r + G(q) = -\Phi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\alpha. \tag{12}$$

In order to design the switching function ψ, consider the generalized Lyapunov function

$$V(t, s) = \tfrac{1}{2} s^T D s. \tag{13}$$

Differentiating (13) with respect to time along the solution of (11) gives

$$\dot{V} = s^T(\Phi\psi + \Phi\alpha - K_d s - Bs) + \frac{1}{2} s^T \left(\frac{d}{dt} D\right) s. \tag{14}$$

Using Property 2, (14) becomes

$$\dot{V} = -s^T K_d s + s^T(\Phi\psi + \Phi\alpha) \tag{15}$$

Now choosing

$$\psi_i = -\bar{\beta}_i \operatorname{sgn}\left(\sum_{j=1}^{n} s_j \Phi_{ji}(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\right), \qquad i = 1 \cdots m \tag{16}$$

where $\bar{\beta}_i \geq |\alpha_i|$ and α_i are defined in (12), are the upper bounds of the unknown parameters, which are assumed to be known. Then

$$\begin{aligned} \dot{V} &= -s^T K_d s - \sum_{i=1}^{m} \bar{\beta}_i \left| \sum_{j=1}^{n} s_j \Phi_{ji} \right| \\ &\quad + \sum_{i=1}^{m} \alpha_i \sum_{j=1}^{n} s_j \Phi_{ji} \\ &\leq -s^T K_d s \\ &\leq -\gamma s^T D s \end{aligned} \tag{17}$$

where $\gamma = \gamma_{\min}[D^{-T/2} K_d D^{-(1/2)}] > 0$ denotes the smallest singular value. From (13), we get

$$\frac{d}{dt} V(t, s) \leq -2\gamma V(t, s) \tag{18}$$

i.e.,

$$V(t, s) \leq V(0, s(0)) e^{-2\gamma t} \tag{19}$$

Thus, the convergence of $\|s\|$ to zero is exponential. Since $\tilde{q}$ and s are related by (8), which, in turn, implies that the tracking error $\|\tilde{q}\|$ will also converge exponentially to zero. Therefore, the following theorem can be obtained.

Theorem 4.1: If the sliding mode control law given by (10) and (16) is applied to the manipulator (1), then in the closed-loop system, the error between the desired and actual trajectory converges to zero exponentially.

The structure of the sliding mode controller given by (10) and (16) is sketched in Fig. 1. The controller consists of two parts. The first part is a special form of dynamics compensation, which attempts to provide the joint dynamic torques necessary to make the desired motions. The second part actually contains two terms representing PD feedback. It intends to regulate the real trajectories about the desired trajectories.

Remark:

1) There are some main differences compared with the adaptive controller described in (6), (7). First, the parameter estimation is not required, avoiding the difficulty linked to the persistency of excitation. Second, $\tilde{q}$ converges exponentially to zero independent of the excitation. Third, the controller assumes that the upper bounds of unknown parameters are available, which is an important clue in guaranteeing the stability of the closed-loop system in VSS design. Also, there will exist the chattering phenomenon in real implementation, which will be discussed later.
2) In sliding mode, the resulting system equation is

$$\dot{\tilde{q}} + \Lambda \tilde{q} = 0. \tag{20}$$

Equation (20) represents n uncoupled first-order linear system and the system only depends on the design parameter $\Lambda > 0$. Clearly, the transient behavior could be prescribed and the robustness to the uncertainties of the system is guaranteed.
3) Compared with literature of VSS control for robots [8]–[18], the main advantage of this algorithm is that the switching gains are determined only in terms of the bounds of robot parameters rather than the bounds of complicated matrix functions of dynamics. These bounds may not be easily obtained because of the complexity of the structure of the matrix functions.
4) It is obvious from (19) that a maximal settling time τ_{s1} is $\tau_{s1} = \ln(V(0)/\epsilon)/2\gamma$, where ϵ is an arbitrarily chosen positive small number. The settling time here means that not later than at the moment $t = \tau_{s1}$ the system tracking trajectory is guaranteed in the ϵ neighborhood of the sliding surface $s = 0$. The total maximal settling time is then

$$\tau_s = \frac{\ln(V(0)/\epsilon)}{2\gamma} + \max_i \frac{\ln(|\tilde{q}_i(t_{s1})|/\epsilon)}{\lambda_M(\Lambda)}$$

where $\lambda_M(\Lambda)$ denotes the largest eigenvalue of the matrix Λ.

B. Robustness with Respect to Uncertainties

In practice, some uncertainties, e.g., the friction coefficients, residual time-varying disturbances, such as stiction or torque ripple, may have some effect on the robot dynamics. The controller must be robust with respect to these uncertainties in the sense that the tracking error for the closed-loop system should be uniformly ultimately bounded [24], [28]. In this section, the behavior of the proposed algorithm (10) and (16) in the presence of uncertainties is analyzed. The dynamics equation (1) becomes

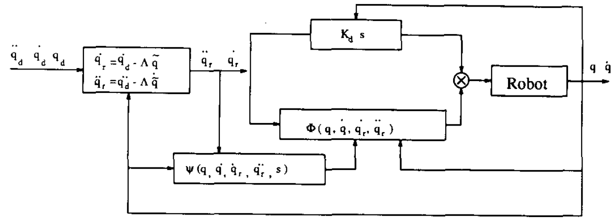

Fig. 1. The structure of the sliding mode controller.

$$D(q)\ddot{q} + B(q, \dot{q})\dot{q} + G(q) + u_\tau = u \tag{21}$$

where D, B, and G are defined in (1), and $u_\tau \in R^n$ is the vector of uncertainties presenting friction, torque disturbance, etc.

In general, in the variable structure system, the uncertainties are assumed to be bounded. This assumption may be reasonable for external disturbance but is rather restrictive as far as unmodeled dynamics are concerned. For example, the viscous and Coulomb friction forces may be modeled as $F_v\dot{q} + F_c \operatorname{sgn}(\dot{q})$. Generally speaking, unmodeled dynamics are functions of the system states and may grow beyond any constant bound if the system becomes unstable. Therefore, it is assumed here that the uncertainty effects are presented by [28]

$$\|u_\tau\| \le d_0 + d_1\|\dot{\tilde{q}}\| + d_2\|\tilde{q}\| \tag{22}$$

where $d_0 > 0$, $d_1 > 0$, and $d_2 > 0$ are some constants.

Concerning the uncertainties, the following theorem is presented.

Theorem 4.2: For the closed-loop system (21), (10), and (16), the sliding variable and tracking error are uniformly bounded if the feedback gain matrix K_d is chosen properly. Furthermore, the ultimate bound of the tracking error is given by

$$\lim_{t \to \infty} \|\tilde{q}\| < \sqrt{\epsilon/2\bar{\gamma}}$$

where $\epsilon = d_0^2/2\lambda_m(K_d)$, $\bar{\gamma} = \min(\bar{\sigma}_d/\lambda_M(D), \bar{\sigma}_\tau/\sigma_\tau)$.

Proof: Combining (10) and (16) with (21) leads to

$$D(q)\ddot{q} + B(q, \dot{q})\dot{q} + G(q) + u_\tau = \Phi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\psi - K_d s.$$

Subtracting (12) from both sides of the above expression, and using the fact that $s = \dot{q} - \dot{q}_r$, one can write

$$D\dot{s} = \Phi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\psi + \Phi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\alpha - K_d s - B(q, \dot{q})s - u_\tau. \tag{23}$$

A Lyapunov function is chosen as

$$V(\bar{e}, t) = \tfrac{1}{2}s^T D s + \tfrac{1}{2}\tilde{q}^T \Gamma \tilde{q} \tag{24}$$

where $\bar{e} = [s, \tilde{q}]^T$ is a generalized error state vector and $\Gamma = \sigma_\tau I$, $\sigma_\tau > 0$ is a constant positive definite matrix.

Differentiating V in (24) with respect to time and using Property 2 give

$$\dot{V} = s^T D\dot{s} + s^T B\dot{s} + \tilde{q}^T \Gamma \dot{\tilde{q}}. \tag{25}$$

Evaluating $\dot{V}$ along the trajectory of (23) yields

$$\begin{aligned}\dot{V} &= s^T\big(\Phi(q,\dot{q},\dot{q}_r,\ddot{q}_r)\psi + \Phi(q,\dot{q},\dot{q}_r,\ddot{q}_r)\alpha \\ &\quad -K_d s - u_\tau\big) + \tilde{q}^T\Gamma\dot{\tilde{q}} \\ &\le -s^T K_d s - s^T u_\tau + \tilde{q}^T \Gamma(s - \Lambda\tilde{q}).\end{aligned} \tag{26}$$

According to (22), one can write

$$\begin{aligned}-s^T u_\tau &\le \|s\|(d_0 + d_1(\|s\| + \Lambda\|\tilde{q}\|) + d_2\|\tilde{q}\|) \\ &\le d_0\|s\| + d_1\|s\|^2 + (\lambda_M(\Lambda)d_1 + d_2)\|s\|\,\|\tilde{q}\|.\end{aligned} \tag{27}$$

It then follows that

$$\begin{aligned}\dot{V} \le &-(\lambda_m(K_d) - d_1)\|s\|^2 \\ &+(\sigma_\tau + \lambda_M(\Lambda)d_1 + d_2)\|s\|\,\|\tilde{q}\| \\ &- \sigma_\tau\lambda_m(\Lambda)\|\tilde{q}\|^2 + d_0\|s\|.\end{aligned} \tag{28}$$

A further manipulation of (28) leads to

$$\begin{aligned}\dot{V} \le &-[\|s\|\,\|\tilde{q}\|]Q\begin{bmatrix}\|s\| \\ \|\tilde{q}\|\end{bmatrix} \\ &- \frac{\lambda_m(K_d)}{2}\left(\|s\| - \frac{d_0}{\lambda_m(K_d)}\right)^2 + \frac{d_0^2}{2\lambda_m(K_d)}\end{aligned} \tag{29}$$

where

$$Q = \begin{bmatrix} \dfrac{\lambda_m(K_d)}{2} - d_1 & -\dfrac{\sigma_\tau + \lambda_M(\Lambda)d_1 + d_2}{2} \\ -\dfrac{\sigma_\tau + \lambda_M(\Lambda)d_1 + d_2}{2} & \sigma_\tau\lambda_m(\Lambda)\end{bmatrix}. \tag{30}$$

It is always possible to properly choose K_d and σ_τ such that $Q > 0$. Therefore, there exists $\bar{\sigma}_d$ and $\bar{\sigma}_\tau$ such that

$$Q = \begin{bmatrix}\bar{\sigma}_d & 0 \\ 0 & \bar{\sigma}_\tau\end{bmatrix} + \tilde{Q} \tag{31}$$

where $\tilde{Q} \ge 0$. Thus

$$\frac{d}{dt}V(\bar{e},t) \le -\bar{\sigma}_d\|s\|^2 - \bar{\sigma}_\tau\|\tilde{q}\|^2 + \frac{d_0^2}{2\lambda_m(K_d)} \tag{32}$$

i.e.,

$$\frac{d}{dt}V(\bar{e},t) \le -2\bar{\gamma}V(\bar{e},t) + \epsilon \tag{33}$$

where $\bar{\gamma} = \min(\bar{\sigma}_d/\lambda_M(D), \bar{\sigma}_\tau/\sigma_\tau)$, $\epsilon = (d_0^2/2\lambda_m(K_d))$.

Thus

$$V(\bar{e},t) \le e^{-2\bar{\gamma}t}\left(V(\bar{e}(0),0) - \frac{\epsilon}{2\bar{\gamma}}\right) + \frac{\epsilon}{2\bar{\gamma}} \tag{34}$$

Therefore

$$\|\tilde{q}\| \le \frac{1}{\sqrt{\sigma_\tau}}e^{-\bar{\gamma}t}\left(V(\bar{e}(0),0) - \frac{\epsilon}{2\bar{\gamma}}\right)^{1/2} + \sqrt{\frac{\epsilon}{2\bar{\gamma}}} \tag{35}$$

$$\lim_{t\to\infty}\|q(t) - q_d(t)\| \le \sqrt{\frac{\epsilon}{2\bar{\gamma}}}.$$

Remark: As we can see, the bound on ϵ can be made arbitrarily small by increasing the control gain K_d, which means increasing the control energy. From a practical point of view, the minimum size of the error bound is limited since sufficiently large control energy may not be available.

C. Smoothing the Control Law

Since the control law (10) and (16) is discontinuous across the sliding surface, such a control law leads to control chattering. Chattering is undesirable in practice because it involves high control activity, and furthermore, may excite unmodeled high-frequency plant dynamics, which could result in unforseen instabilities. This has been recognized by workers in the field, e.g., [9], [10], [16], [22], who have suggested modifications in the control law to overcome the difficulties encountered. This takes the form of using $x/(|x| + \delta)$ in the place of $\mathrm{sgn}(x)$ in the control law (10), where δ is a constant. However, in such a case, $\tilde{q}$ does not tend to zero but is uniformly bounded. In the following analysis, it is shown that the admissible amplitude of tracking error, given by engineering consideration, can be achieved by choosing a suitable δ.

Let the switching function ψ in the control law (10) be replaced by

$$\psi_i = -\hat{\beta}_i\frac{\varphi_i}{|\varphi_i| + \delta_i} \qquad i = 1\cdots m \tag{36}$$

$$\hat{\beta}_i \ge \left(1 + \frac{\delta_i}{\epsilon_i}\right)|\alpha_i| \qquad i = 1\cdots m \tag{37}$$

where $\varphi_i \triangleq (\sum_{j=1}^n s_j\Phi_{ji}(q,\dot{q},\dot{q}_r,\ddot{q}_r))$, $\delta_i > 0$ and $\epsilon_i > 0$ are arbitrary constants, α_i are defined in (12).

Such chosen control law (10), (36), and (37) leads to the following theorem.

Theorem 4.3: Set $\epsilon_i(> 0)$ and $\delta_i(> 0)$ arbitrarily. Then, the control system defined by (1), (10), (36), and (37) is uniformly bounded, and the tracking error $\tilde{q}$ converges to a region described by $S(\delta) = \{\tilde{q}; \|s\| < \sqrt{\bar{\epsilon}/2\gamma}\}$, where $\bar{\epsilon} = \sum_{i=1}^m|\alpha_i|\delta_i$, $\gamma = \gamma_{\min}[D^{-(T/2)}K_dD^{-(1/2)}]$, and α_i is defined in (12).

Proof: Consider the generalized Lyapunov function given in (13). Using the same derivation in the proof of

Theorem 4.1, leads to

$$\dot{V} = -s^T K_d s + s^T(\Phi\psi + \Phi\alpha). \tag{38}$$

In the case where $|\varphi_i| \geq \epsilon_i$, (38) becomes

$$\dot{V} \leq -s^T K_d s - \sum_{i=1}^{m} \hat{\beta}_i \frac{\varphi_i^2}{|\varphi_i| + \delta_i} + \sum_{i=1}^{m} |\alpha_i| |\varphi_i| \leq -s^T K_d s \leq 0. \tag{39}$$

On the other hand, in case that $|\varphi_i| < \epsilon_i$, (38) becomes

$$\frac{d}{dt} V(s,t) \leq -2\gamma V(s,t) + \bar{\epsilon} \tag{40}$$

where $\gamma = \gamma_{\min}[D^{-(T/2)} K_d D^{-(1/2)}]$, $\bar{\epsilon} = \sum_{i=1}^{m} |\alpha_i| \delta_i$. Thus

$$V(s,t) \leq e^{-2\gamma t}\left(V(s(0),0) - \frac{\bar{\epsilon}}{2\gamma}\right) + \frac{\bar{\epsilon}}{2\gamma}. \tag{41}$$

Therefore

$$\|s\| \leq e^{-\gamma t}\left(V(s(0),0) - \frac{\bar{\epsilon}}{2\gamma}\right)^{1/2} + \sqrt{\frac{\bar{\epsilon}}{2\gamma}}$$

$$\lim_{t\to\infty} \|s(t)\| \leq \sqrt{\frac{\bar{\epsilon}}{2\gamma}}. \tag{42}$$

Remark:

1) The two constant parameters ϵ_i and δ_i $(i = 1,\cdots,m)$ are introduced. δ_i may be considered to be the difference from an ideal discontinuous control input and the ratio δ_i/ϵ_i determines the magnitudes of control gains. The optimal selections of ϵ_i, δ_i are worthy of further investigation.
2) Using the same argument as in Section III-B, the smoothed control law (10), (36), and (37) retains robust with respect to uncertainties. The detailed discussions are not given here in order to save space.

V. Real-Time Implementation

A. Description of Control System

To demonstrate the validity of the proposed robust algorithm (10) and (16), a real-time implementation of the control strategy was developed for only a two-degree of freedom out of a self-built five-axis manipulator. Since a robotic manipulator must have a three degree-of-freedom, at least in order to move to an arbitrary point in space, a two degree-of-freedom system, however, is sufficient to examine the validity of the control strategy.

The controlled two-linkage manipulator is shown in Fig. 2. All of its physical parameters are given in Table I. A dc servomotor is mounted on each joint, and coupled the links through harmonic drives with the gear ratio being 1:60. The characteristics of actuators are shown in the Table II. The computer controller is a Motorola M68000 16-b single-board microprocessor (SBM) running at 8 MHz. The SBM provides voltages to the PWM power amplifiers through 12-b D/A converters and the robot motors are fed by PWM power amplifiers. The manipulator angular positions were fed back to the microprocessor from encoders mounted at the motor shafts. The encoder outputs were converted into a count representing angular positions and read by the microprocessor through a 16-b parallel port. The speed information was derived from the encoder output at sampling instants. The hardware details of the control system are shown in Fig. 3.

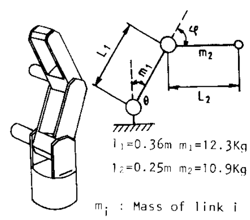

Fig. 2. Two-linkage manipulator.

Software for implementing the control algorithm was developed in PASCAL programming language together with M68000 Assembly language. The impetus for using PASCAL was primarily due to the availability of a PASCAL compiler for M68000 and the complexity of the control strategy. Inasmuch as the SBM does not have a floating point processor, all floating point arithmetic operations are performed in software, and therefore a significant performance bottleneck is caused. Thus the choice of sample time is restricted, and the sample interval cannot be selected to be small enough. Hence, in this situation, the desired trajectory to be tracked may not be planned too quickly and the switching frequency, which should be ideally infinite, is limited by the microprocessor speed.

B. Dynamic model and Controller Design

The dynamics of the actuators can be approximately described by [27]

$$h_{11}\ddot{\theta}_1 + h_{21}\dot{\theta}_1 + h_{31}u_1 = v_1 \tag{43}$$

$$h_{12}\ddot{\theta}_2 + h_{22}\dot{\theta}_2 + h_{32}u_2 = v_2 \tag{44}$$

where h_{ij} $(i = 1,2,3\ j = 1,2)$, as shown in Table III, are known constants; v_i $(i = 1,2)$ are inputs of actuators.

Therefore, the dynamics of the two-link manipulator, including actuators, are described in [15], [27]:

$$\begin{bmatrix} \alpha + \beta + 2\eta\cos\theta_2 & \eta\cos\theta_2 + \beta \\ \eta\cos\theta_2 + \beta & \beta \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -\eta\dot{\theta}_2\sin\theta_2 & -(\dot{\theta}_1 + \dot{\theta}_2)\eta\sin\theta_2 \\ \eta\dot{\theta}_1\sin\theta_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} \alpha\cos\theta_1 + \eta\cos(\theta_1 + \theta_2) \\ \eta\cos(\theta_1 + \theta_2) \end{bmatrix} \frac{g}{l_1} = \begin{bmatrix} v_1/h_{31} \\ v_2/h_{32} \end{bmatrix} - \begin{bmatrix} h_{21}/h_{31} & 0 \\ 0 & h_{22}/h_{32} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{45}$$

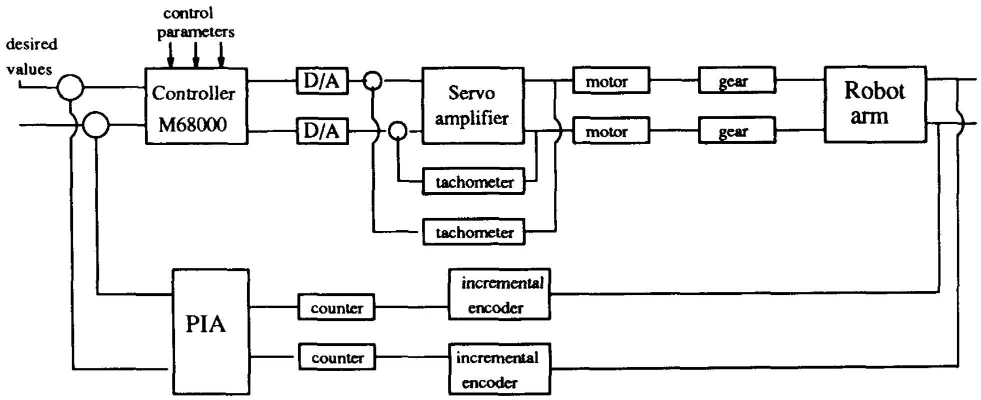

Fig. 3. Schematic diagram of robot control system.

TABLE I
PHYSICAL PARAMETERS OF TWO-LINKAGE ARM

Parameters	Lower	Upper
Link inertia I_i (kg.m)	0.09599	0.04115
Link mass m_i (kg)	12.3	10.9
Link length L_i	0.36	0.25
Position of mass center l_i (m)	0.18	0.12

TABLE II
PARAMETERS FOR ACTUATORS

Rated voltage	120 V
No load speed at rated voltage	3750 r/min
Torque constant	43 ozin/A
Voltage constant	32 V/kr/min
Armature moment of inertial	0.03 ozins
Armature electric time const.	3.1 ms
Armature mechanical time const.	15 ms
Tachometer voltage gradient	30 V/kr/min

TABLE III
KNOWN CONSTANTS

$h_{11} = 0.042$	$h_{12} = 0.033$	$h_{21} = 3.5$
$h_{22} = 3.04$	$h_{31} = 0.027$	$h_{32} = 0.016$

where l_1 is a known constant, g is the acceleration of gravity, and three unknown parameters α, β, and η are functions of unknown physical parameters of the manipulator.

In order to examine the validity of the proposed method, the manipulator is required to move along the desired trajectories. The desired trajectories, illustrated in Fig. 4, are planned by using the 4-3-4 joint-interpolated method. The sliding surfaces are chosen as

$$s_1 = \sigma_s(\theta_1 - \theta_{1d}) + \left(\dot{\theta}_1 - \dot{\theta}_{1d}\right)$$

$$s_2 = \sigma_s(\theta_2 - \theta_{2d}) + \left(\dot{\theta}_2 - \dot{\theta}_{2d}\right)$$

where θ_{1d} and θ_{2d} are given desired trajectories. The resulting sliding mode equations are two decoupled first-order systems

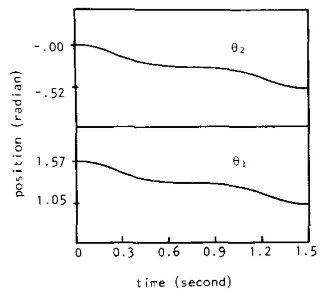

Fig. 4. Desired trajectories.

$$\left(\dot{\theta}_i - \dot{\theta}_{id}\right) = -\sigma_s(\theta_i - \theta_{id}) \qquad i = 1,2.$$

For simplicity, the feedback gain matrix K_d is chosen to be diagonal $K_d = \text{diag}(\sigma_d, \sigma_d)$. The regressor matrix $\Phi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ used in (10) can be expressed as

$$\Phi_{11} = \ddot{\theta}_{1r} + e\cos(\theta_2)$$

$$\Phi_{12} = \ddot{\theta}_{1r} + \ddot{\theta}_{2r}$$

$$\Phi_{13} = 2\ddot{\theta}_{1r}\cos(\theta_2) + \ddot{\theta}_{2r}\cos(\phi) - \dot{\theta}_2\dot{\theta}_{1r}\sin(\theta_2) - \left(\dot{\theta}_1 + \dot{\theta}_2\right)\dot{\theta}_{2r}\sin(\theta_2) + e\cos(\theta_1 + \theta_2)$$

$$\Phi_{21} = 0$$

$$\Phi_{22} = \Phi_{12}$$

$$\Phi_{23} = \dot{\theta}_1\dot{\theta}_{1r}\sin(\theta_2) + \ddot{\theta}_{1r}\cos(\theta_2) + e\cos(\theta_1 + \theta_2)$$

where $\dot{\theta}_{1r} = \dot{\theta}_{1d} - (\theta_1 - \theta_{1d})$, $\dot{\theta}_{2r} = \dot{\theta}_{2d} - (\theta_2 - \theta_{2d})$, $e = g/l_1$.

A removable 2-kg load was placed on the end of the manipulator. Test runs were made both with and without this load. Changes in the load were not accounted for in the controller in order to test the robustness of the controller. The smoothing controller (10), (36), and (37) was run at a sample interval of 0.015 s; the parameters of the controller are given in Table IV.

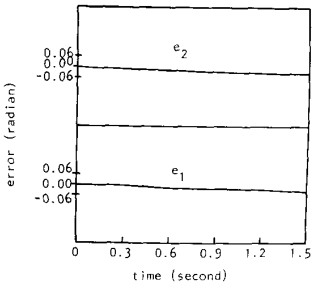

Fig. 5. Tracking errors.

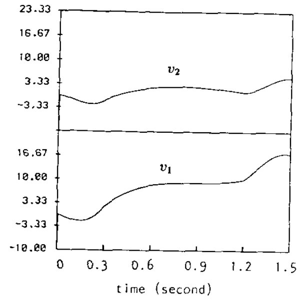

Fig. 6. Control inputs.

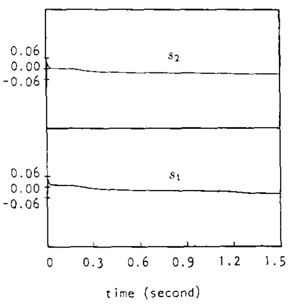

Fig. 7. Sliding variables s_1 and s_2.

TABLE IV
PARAMETERS OF THE CONTROLLER

σ_s $(\Lambda = \sigma_s I) = 5$	σ_d $(K_d = \sigma_d I) = 5$	$\delta_1 = 0.05$
$\delta_2 = 0.05$	$\delta_3 = 0.05$	$\hat{\beta}_1 = 4$
$\hat{\beta}_2 = 1$	$\hat{\beta}_3 = 1$	$\epsilon_3 = 0.1$
$\epsilon_1 = 0.1$	$\epsilon_2 = 0.1$	

C. Experimental Results

System time responses were obtained from actual measurement and stored in the SBM, displayed on a CRT, and recorded. The experimental results for the maneuver as described in (10), (36), and (37) with the load attached are shown in Figs. 5–7. Fig. 5 shows the trajectory tracking errors. Fig. 6 shows the control inputs, and the sliding variable s are shown in Fig. 7. Fig. 8 shows the joint angle tracking errors with the load removed from the end of link 2. Fig. 9 shows the control input without the load. It is confirmed that validity of this novel variable structure controller is used explicitly for the purpose of trajectory tracking in the presence of uncertainties of the system.

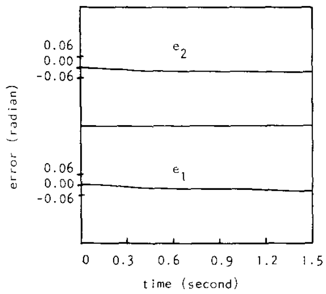

Fig. 8. Tracking errors without load.

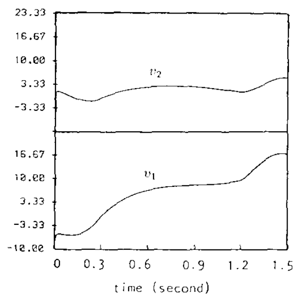

Fig. 9. Control inputs without load.

VI. CONCLUSION

A novel variable structure control scheme has been developed using the theory of the variable structure systems for the trajectory control of robot manipulators. Stability and robustness in the presence of uncertainties are analysed and discussed. The response transient is at least of the exponential type, with a decay rate larger than a certain value, independent of the excitation signal (no persistency of excitation is required). Problems inherent to the integral adaptation law such as the parameter drift do not appear in the scheme. A smoothed control law is also suggested, which overcomes principal drawbacks of the variable structure method. It is shown that the smoothed control renders the closed-loop system globally uniformly ultimately bounded with respect to a set $S(\delta)$, which can be made arbitrarily small by decreasing δ. The proposed algorithm was implemented on a Motorola M68000 single-board microprocessor interfaced to a two degree-of-freedom manipulator. The experimental results show the good tracking of the manipulator with desired trajectories in the presence of such uncertainty as handling a varying payload.

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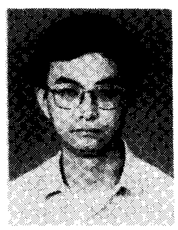

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