Hybrid Adaptive/Robust Motion Control of Rigid-Link Electrically-Driven Robot Manipulators

Chun-Yi Su, *Member, IEEE,* and Yury Stepanenko

***Abstract*—In this paper a hybrid adaptive/robust control scheme is proposed for rigid-link electrically-driven robot manipulators in the presence of arbitrary uncertain inertia parameters of the manipulator and the electrical parameters of the actuators. In contrast to the known methods, the presented design requires at most the joint velocity feedback and does not rely on the knowledge of the bounds of complexity functions. Semi-global asymptotic stability of the adaptive/robust controller is established in the Lyapunov sense. Simulation results are included to demonstrate the tracking performance.**

I. Introduction

Various control methods have been developed in the literature for rigid robot motion control. The interested reader is referred to Abdallah *et al.* [1] and Ortega and Spong [11] for recent reviews. The principal limitation associated with many of these schemes is that controllers are designed at torque input level and actuator dynamics are excluded. However, as demonstrated by Good *et al.* [7], the actuator dynamics constitute an important part of the complete robot dynamics, especially in the cases of high-velocity movements and highly varying loads. The inclusion of the actuators into the dynamic equations complicates the controller structure and its stability analysis since the systems are described by third-order differential equations [15].

The study of the control of rigid robots including the actuators was an open problem until recent efforts described in [2], [3], [5], [6], [8], [10], [15], [16], [20]. Based on the Freund's nonlinear control theory, Beekmann and Lee [2] proposed a nonlinear control law. By using the linearizable method, Taylor [16] presented a control method where the switched reluctance motor was considered as the actuators, and Tarn *et al.* [15] developed a controller with direct-current motors as actuators. But it should be noted that the design procedure in the aforementioned schemes is based on the full knowledge of the complex dynamics of robotic systems. If there are uncertainties in the system dynamics, controllers so designed may give degraded performance and may incur instability. The schemes given in [6] and [14] only dealt with the uncertainty in the manipulator and require the full knowledge of the actuator parameters. To deal with the uncertainties in the combined dynamics, some promising robust schemes were recently proposed in [3], [5], [8], [10].

The objective of this study is similar to that in [3], [5], [8]. A hybrid adaptive/robust control law is proposed for n-link manipulators which include the effects of actuator dynamics. The proposed controller has the following features: it does not require the joint acceleration feedback and the knowledge of the bounds of complexity functions (the derivative of *fictitious* forces). Asymptotic stability of the adaptive controller is established in the Lyapunov sense.

The arrangement of this article is as follows: in Section II the robot dynamics including actuators is expressed in the form of two cascaded loops: the actuator loop and the manipulator loop. A desired *fictitious* force is introduced as a synthesized input signal intended for the manipulator loop. A *corrective* control law is then used for the usually neglected electrical actuator loop. Asymptotic stability of the adaptive controller is established in the Lyapunov sense. Note that the terms *fictitious* and *corrective* come from [5]. In Section III a simplified algorithm is introduced to avoid the calculation of the derivative of the regressor matrix. Simulation results are discussed in Section IV. Finally some conclusions are given in Section V.

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The authors are with the Department of Mechanical Engineering, University of Victoria, Victoria, B.C. V8W 3P6, Canada.

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II. Derivation of the Control Law

Consider an n-link manipulator with revolute joints driven by armature-controlled dc motors with voltages being inputs to amplifiers. As in [5], [8], [15], the dynamics are described by

$$(D(\boldsymbol{q}) + J)\ddot{\boldsymbol{q}} + B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) = K_N \boldsymbol{I} \quad (1)$$

$$L\dot{\boldsymbol{I}} + R\boldsymbol{I} + K_e\dot{\boldsymbol{q}} = \boldsymbol{u} \quad (2)$$

where $\boldsymbol{q} \in R^n$ is the vector of the joint position, $\boldsymbol{I} \in R^n$ is the vector of the armature currents and $\boldsymbol{u} \in R^n$ is the vector of the armature voltages; $D(\boldsymbol{q})$ is the manipulator mass-matrix, which is a symmetric positive definite matrix; $B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ represents the centripetal and Coriolis force; $G(\boldsymbol{q})$ denotes the gravitational force; J is the actuator inertia matrix; L represents the actuator inductance matrix; R is the actuator resistance matrix, K_e is the matrix characterizing the voltage constant of the actuator and K_N is the matrix which characterizes the electromechanical conversion between current and torque. While $D(\boldsymbol{q}), B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ and $G(\boldsymbol{q})$ are nonlinear functions, J, L, R, K_e and K_N are positive definite constant diagonal matrices. We note only that the matrix $(\dot{D} - 2B)$ is a skew-symmetric matrix.

It is assumed that $\dot{\boldsymbol{q}}$, $\boldsymbol{q}$ and $\boldsymbol{I}$ are measurable and the exact values of the robotic functions $D(\boldsymbol{q}), B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ and $G(\boldsymbol{q})$ and actuator dynamic coefficient matrices J, L, R, K_e and K_N are not available. The considered adaptive controller design problem is as follows: For any given desired bounded trajectories $\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d$, and $\boldsymbol{q}_d^{(3)} \in R^n$, with some or all of the manipulator parameters unknown, derive a controller for the actuator voltages $\boldsymbol{u}$ such that the manipulator position vector $\boldsymbol{q}(t)$ tracks $\boldsymbol{q}_d(t)$.

The dynamic model (1) and (2) consists actually of two cascaded loops. Unlike the dynamic model of a robot at the torque input level, the virtual force $K_N\boldsymbol{I}$ in (1) cannot be synthesized directly. Instead, it is the output of the actuator dynamics. In accordance with the cascade control strategy described by [5], [8], [18], the design procedure is organized as a two-step process. Firstly, the vector $\boldsymbol{I}$ is regarded as a control variable for subsystem (1) and a control input $\boldsymbol{I}_d$ is designed so that the tracking goal may be achieved. Secondly, $\boldsymbol{u}$ is designed such that $\boldsymbol{I}$ tracks $\boldsymbol{I}_d$. In turn, this allows $\boldsymbol{q}(t)$ to track $\boldsymbol{q}_d(t)$. In this paper (1) is called the *manipulator loop* and (2) the *actuator loop.*

A. Adaptive Control for the Manipulator Loop

Using the desired armature current vector $\boldsymbol{I}_d$, the model (1) can be rewritten as

$$(D(\boldsymbol{q}) + J)\ddot{\boldsymbol{q}} + B(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) = K_N\boldsymbol{I}_d + K_N\tilde{\boldsymbol{I}} \quad (3)$$

where $\tilde{\boldsymbol{I}} \triangleq \boldsymbol{I} - \boldsymbol{I}_d$ represents a current perturbating to the rigid-link dynamics. The system (1) can be viewed as a rigid model system with an input disturbance $K_N\tilde{\boldsymbol{I}}$, controlled by $K_N\boldsymbol{I}_d$. The synthesis of $K_N\boldsymbol{I}_d$ may follow any available design procedures developed at the torque input level.

However, the direct application of design procedures developed at the torque input level to design $\boldsymbol{I}_d$ is impaired by the assumption that the electromechanical conversion matrix K_N is not exactly available, and thus $\boldsymbol{I}_d$ cannot be calculated from $K_N\boldsymbol{I}_d$. Therefore, one needs a modified scheme in order to directly generate the signal $\boldsymbol{I}_d$.

In order to solve this problem, firstly, based on the parameterization technique as in [11], the nonlinear terms D, B, and G in (1) can be expressed as

$$(D(\boldsymbol{q})+J)\ddot{\boldsymbol{q}}_d + B(\boldsymbol{q},\dot{\boldsymbol{q}}_d)\dot{\boldsymbol{q}}_d + G(\boldsymbol{q}) = \Phi(\boldsymbol{q},\dot{\boldsymbol{q}}_d,\ddot{\boldsymbol{q}}_d)\alpha \tag{4}$$

where $\Phi(\boldsymbol{q},\dot{\boldsymbol{q}}_d,\ddot{\boldsymbol{q}}_d) \in R^{n\times m}$ is the regressor matrix independent of the dynamic parameters, α is a constant vector of manipulator inertia parameters.

As in [8], let Φ be written as $\Phi^T = [\phi_1\phi_2\cdots\phi_n]$, where ϕ_i^T is the ith row of Φ, and introduce the augmented regressor matrix $\Phi_a(\boldsymbol{q},\dot{\boldsymbol{q}}_d,\ddot{\boldsymbol{q}}_d)$ defined as

$$\Phi_a(\boldsymbol{q},\dot{\boldsymbol{q}}_d,\ddot{\boldsymbol{q}}_d) \triangleq \begin{bmatrix} \phi_1^T & 0 & \cdot & 0 & 0 \\ 0 & \phi_2^T & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 & \phi_n^T \end{bmatrix},$$

then

$$K_N^{-1}\Phi\alpha = K_N^{-1}\Phi_a\alpha_a = \Phi_a K_{Na}^{-1}\alpha_a = \Phi_a\alpha_{ak} \tag{5}$$

where $K_{Na} \triangleq \mathrm{diag}[k_{Ni}I_m], \alpha_a^T \triangleq [\alpha^T\alpha^T\cdots\alpha^T]$ is a corresponding augmented inertia parameter vector, $\alpha_{ak}^T \triangleq [k_{N1}^{-1}\alpha^T k_{N2}^{-1}\alpha^T\cdots k_{Nn}^{-1}\alpha^T]$.

We suppose only that the parameter vector α_{ak} is "*uncertain*". Following the results of [18], the desired $\boldsymbol{I}_d$ is then synthesized by

$$\boldsymbol{I}_d = \Phi_a(\boldsymbol{q},\dot{\boldsymbol{q}}_d,\ddot{\boldsymbol{q}}_d)\hat{\alpha}_{ak} - \gamma^2\Gamma(\boldsymbol{w}+\kappa\tilde{\boldsymbol{q}}) \tag{6}$$

where $\tilde{\boldsymbol{q}} \triangleq \boldsymbol{q}-\boldsymbol{q}_d$ is the joint tracking error; Γ is an arbitrary positive definite constant diagonal matrix; γ and κ are positive constants; $\boldsymbol{w}$ is an intermediate vector synthesized by

$$\dot{\boldsymbol{w}} = -2\gamma\boldsymbol{w} + \gamma^2\dot{\tilde{\boldsymbol{q}}}. \tag{7}$$

The adaptive law for adjusting $\hat{\alpha}_{ak}$ is given by

$$\dot{\tilde{\alpha}}_{ak} = \dot{\hat{\alpha}}_{ak} = -\sigma\Phi_a^T z \tag{8}$$

$$z \triangleq \dot{\tilde{\boldsymbol{q}}} - \frac{1}{\gamma}\boldsymbol{w} + \frac{\kappa}{\gamma}\tilde{\boldsymbol{q}} \tag{9}$$

where $\tilde{\alpha}_{ak} \triangleq \hat{\alpha}_{ak} - \alpha_{ak}$ denotes the parameter error vector.

It should be mentioned that $\boldsymbol{I}_d$ given by the control law (6) and (7) and adaptive law (8) and (9) does not involve velocity feedback $\dot{\boldsymbol{q}}$. This fact will be used later to prove that the controller of the overall system will only depend on measurements of I, q and $\dot{q}$.

Substituting (6) into (3), one obtains the joint position error equation:

$$\begin{aligned} K_N^{-1}(D(\boldsymbol{q})+J)\ddot{\tilde{\boldsymbol{q}}} = &-\gamma^2\Gamma\boldsymbol{w} - \kappa\gamma^2\Gamma\tilde{\boldsymbol{q}} + \tilde{\boldsymbol{I}} - K_N^{-1}B(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} \\ &- K_N^{-1}B_d\dot{\tilde{\boldsymbol{q}}} + \Phi_a\tilde{\alpha}_{ak} \end{aligned} \tag{10}$$

where $B_d\dot{\tilde{\boldsymbol{q}}} \triangleq B(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_d - B(\boldsymbol{q},\dot{\boldsymbol{q}}_d)\dot{\boldsymbol{q}}_d$. It can be shown that B_d is an uniformly bounded matrix because $\dot{\boldsymbol{q}}_d$ is uniformly bounded.

Introducing a state vector $\boldsymbol{x}^T \triangleq [\dot{\tilde{\boldsymbol{q}}}^T, \boldsymbol{w}^T, \tilde{\boldsymbol{q}}^T]$, then the dynamic (10) can be expressed in state space as

$$\dot{\boldsymbol{x}} = -A\boldsymbol{x} + C(\tilde{\boldsymbol{I}} - K_N^{-1}B(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} - K_N^{-1}B_d\dot{\tilde{\boldsymbol{q}}} + \Phi_a\tilde{\alpha}_{ak}) \tag{11}$$

where (7) is incorporated to obtain

$$A \triangleq \begin{bmatrix} 0 & \gamma^2(D+J)^{-1}K_N\Gamma & \kappa\gamma^2(D+J)^{-1}K_N\Gamma \\ -\gamma^2 I & 2\gamma I & 0 \\ -I & 0 & 0 \end{bmatrix}$$

and

$$C \triangleq \begin{bmatrix} (D+J)^{-1}K_N \\ 0 \\ 0 \end{bmatrix}.$$

Following the argument of [18], an important design procedure is to find a pair of positive definite matrices P and Q such that $1/2(PA + A^TP) = Q$. A possible choice is given by

$$P \triangleq \begin{bmatrix} (D+J) & \frac{-1}{\gamma}(D+J) & \frac{\kappa}{\gamma}(D+J) \\ \frac{-1}{\gamma}(D+J) & K_N\Gamma & 0 \\ \frac{\kappa}{\gamma}(D+J) & 0 & \kappa\gamma^2 K_N\Gamma \end{bmatrix}$$

and

$$Q \triangleq \begin{bmatrix} (\gamma - \frac{\kappa}{\gamma})(D+J) & -(D+J) & 0 \\ -(D+J) & \gamma K_N\Gamma & 0 \\ 0 & 0 & \kappa^2\gamma K_N\Gamma \end{bmatrix}.$$

Both P and Q are positive definite if γ is sufficiently large. Their eigenvalues satisfy the following bounds:

$$\lambda_p \le \inf_{\|\boldsymbol{x}\|=1}\boldsymbol{x}^TP\boldsymbol{x} \quad \text{and} \quad \gamma\lambda_q \le \inf_{\|\boldsymbol{x}\|=1}\boldsymbol{x}^TQ\boldsymbol{x}. \tag{12}$$

Before the introduction of the control law of the actuator loop which compensates the disturbance $K_N\tilde{\boldsymbol{I}}$, it is helpful to study the closed-loop system stability of the manipulator loop when $\tilde{\boldsymbol{I}}$ is zero. The closed-loop system is described by (8) and (11). Its asymptotic stability is established by the following lemma.

Lemma: The closed-loop system described by (8) and (11) is asymptotically stable if $\tilde{\boldsymbol{I}} = 0$ and γ initially satisfies

$$\gamma\lambda_q > 3\|B_d\| + 2\vartheta\left(\sup\|\dot{\boldsymbol{q}}_d\| + \sqrt{\frac{2}{\lambda_p}L_a}\right), \tag{13}$$

where λ_p and λ_q are defined in (12); L_a is a function defined in (28).

Proof: See Appendix A.

B. Hybrid Adaptive/Robust Control for the Cascade Control System

We can now use (6) to design a control law at the voltage input $\boldsymbol{u}$, which forces $\tilde{\boldsymbol{I}}$ to zero. In the following development we suppose that the electrical parameters K_N, L, R, and K_e are all of uncertain values. However, there exist L_0, R_0, and K_{eo}, all known, such that

$$\|L - L_0\| \le \delta_1; \quad \|R - R_0\| \le \delta_2; \quad \|K - K_{eo}\| \le \delta_3 \tag{14}$$

With the above in mind, the robust *corrective* control law, forcing $\tilde{\boldsymbol{I}} = 0$, is then synthesized by

$$\begin{aligned} \boldsymbol{u} = &L_0\dot{\boldsymbol{I}}_d + R_0\boldsymbol{I}_d + K_{eo}\dot{\boldsymbol{q}}_d \\ &- (\hat{\delta}_1\|\dot{\boldsymbol{I}}_d\| + \hat{\delta}_2\|\boldsymbol{I}_d\| + \hat{\delta}_3\|\dot{\boldsymbol{q}}_d\|)\,\mathrm{sgn}(\tilde{\boldsymbol{I}}) \end{aligned} \tag{15}$$

$$\dot{\hat{\delta}}_1 = \eta_1\|\dot{\boldsymbol{I}}_d\|\|\tilde{\boldsymbol{I}}\| \tag{16}$$

$$\dot{\hat{\delta}}_2 = \eta_2\|\boldsymbol{I}_d\|\|\tilde{\boldsymbol{I}}\| \tag{17}$$

$$\dot{\hat{\delta}}_3 = \eta_3\|\dot{\boldsymbol{q}}_d\|\|\tilde{\boldsymbol{I}}\| \tag{18}$$

where $\boldsymbol{I}_d$ is defined in (6); $\eta_i (i = 1, 2, 3)$ are constants, determining the rates of the adaptations. As in [5], the term *corrective* control is used to highlight the fact that the robust adaptive control law given in (15)–(18) corrects for the usually neglected electrical actuator dynamics.

The structure of the controller given by (15) is sketched in Fig. 1. The controller consists of two parts. In the first part $\boldsymbol{I}_d$ represent a *fictitious* control input, which may be viewed as a adaptive controller that ensures the convergence of the tracking error if the actuator dynamics were not present. In the second part the

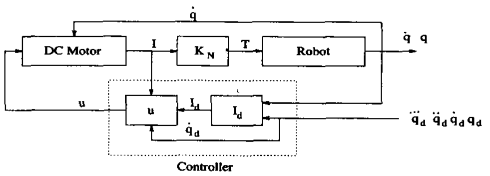

Fig. 1. Control system.

input voltage $\boldsymbol{u}$ intends to regulate the real armature currents about the *fictitious* currents and therefore attempts to provide the control voltages necessary to make the desired motions.

The stability of the closed-loop system described by (1), (2), (6) and (15) is established in the following theorem.

Theorem 1: If the robust control voltages $\boldsymbol{u}$ given by (6) and (15) are applied to the manipulator (1), (2), then in the closed-loop system the vectors $\tilde{\boldsymbol{q}}$ and $\tilde{\boldsymbol{I}}$ are both asymptotically stable, provided γ initially satisfies

$$\gamma\lambda_q > 3\|B_d\| + \mu + 2\vartheta\left(\sup\|\dot{\boldsymbol{q}}_d\| + \sqrt{\frac{2}{\lambda_p}V}\right), \tag{19}$$

where λ_p and λ_q are defined in (12); V is a function defined in (33) while

$$\mu \triangleq \frac{\beta^2}{4\lambda_r}, \quad \beta = (3 + \|K_e\|) \quad \text{and} \quad \lambda_r \triangleq \inf \frac{\tilde{\boldsymbol{I}}^T R \tilde{\boldsymbol{I}}}{\|\tilde{\boldsymbol{I}}\|^2}.$$

Proof: See Appendix B.

Remarks:

1) It should be noted that the control law given by (15)–(18) depends on the calculation of $\dot{\boldsymbol{I}}_d$. Since $\boldsymbol{I}_d$ in (6) only involves the position feedback $\boldsymbol{q}$, the derivative of $\boldsymbol{I}_d$, therefore, only needs velocity feedback $\dot{\boldsymbol{q}}$. This is the motivation of synthesizing $\boldsymbol{I}_d$ in (6). Actually, the development of $\boldsymbol{I}_d$ is based on the *lead-plus-bias* controller proposed in [19] such that the velocity feedback is avoided. In this case, the adaptive control law (15)–(18) for the cascade control system only requires the measurements of $\boldsymbol{I}, \boldsymbol{q}$ and $\dot{\boldsymbol{q}}$.
2) There is a nontrivial difference between the adaptive law (15) and the control laws in [5], [8], which also only require the measurements of $\boldsymbol{I}, \boldsymbol{q}$ and $\dot{\boldsymbol{q}}$. Since the fictitious control laws for manipulator loop developed in [5], [8] involve the feedbacks of $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$, the knowledge of the bounds of the derivative of fictitious control laws is needed to avoid the requirement of acceleration feedback $\ddot{\boldsymbol{q}}$. In contrast, the bound of $\dot{\boldsymbol{I}}_d$ is not required in our scheme.
3) Compared with the adaptive scheme [3], the proposed scheme utilize the sliding mode method for the *corrective* control in the actuator loop, in which as in [4] the adaptive laws adjust control gains directly without estimating the unknown actuator parameters. The controller so developed is structurally simple as well as computationally fast. However, the sliding mode method is actually a high gain scheme which may result in chattering behavior. If a boundary layer is used to eliminate the chattering, as discussed below, only uniformly ultimately bounded tracking errors can be guaranteed as opposed to [3]. Therefore, the trade-off should be made between the simplicity and control accuracy.
4) The control law (15) involves the discontinuous functions and may result in chattering behavior. However, in this case, the chattering signal is the voltage. As demonstrated in [12], the torque signal is continuous after the low-pass filtering of the motor dynamics. From a practical point of view, a chattering voltage is less difficult to synthesize and less prohibitive than a chattering torque, since many DC motors are controlled by pulse-width modulation (PWM) signals. If the chattering effects should be eliminated, it can be done by introducing a boundary layer at a sacrificed control accuracy. In our adaptive scheme, it is easy to replace sgn($\tilde{\boldsymbol{I}}$) in (15) by

$$\pi(\tilde{\boldsymbol{I}}) = \begin{cases} \operatorname{sgn}(\tilde{\boldsymbol{I}}) & \text{if } \tilde{\boldsymbol{I}} > \varepsilon \\ \tilde{\boldsymbol{I}}/\varepsilon & \text{if } \tilde{\boldsymbol{I}} \leq \varepsilon \end{cases}$$

for some small $\varepsilon > 0$. However, the stability result changes. It can only be shown that closed-loop system is uniformly ultimately bounded.
5) In this paper, the bounds on δ_1, δ_2 and δ_3 are not assumed to be available and suitable integral updated laws are given so that $\hat{\delta}_1, \hat{\delta}_2$ and $\hat{\delta}_3$ grow until they reach to whatever levels are necessary to cancel the nonlinear dynamics.

III. A Simplified Algorithm

From (15) we require to calculate

$$\dot{\boldsymbol{I}}_d = (d/dt)(\Phi_a(\boldsymbol{q}, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\hat{\alpha}_{ak}) - \gamma^2\Gamma(\dot{\boldsymbol{w}} + \kappa\dot{\tilde{\boldsymbol{q}}})$$

where $(d/dt)(\Phi_a\hat{\alpha}_{ak}) = \dot{\Phi}_a\hat{\alpha}_{ak} + \Phi_a\dot{\hat{\alpha}}_{ak}$. The computation of $\dot{\Phi}_a$ may be challenging. It seems that there are no reports on how to *recursively* compute it for a general n-link manipulator in the literature. If such an algorithm were developed, it might be computational expensive to update $\dot{\Phi}_a$. In order to avoid the intensive computation of $\dot{\Phi}_a$, we simply substitute

$$\dot{\boldsymbol{I}}_m \triangleq -\gamma^2\Gamma(\dot{\boldsymbol{w}} + \kappa\dot{\tilde{\boldsymbol{q}}}) \tag{20}$$

for $\dot{\boldsymbol{I}}_d$ since the feedback signal $\boldsymbol{I}_m = -\gamma^2\Gamma(\boldsymbol{w} + \kappa\tilde{\boldsymbol{q}})$ pays a vital role in the stability of the closed-loop system whereas the effect of the feedforward signal $\boldsymbol{I}_f \triangleq \Phi_a(\boldsymbol{q}, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)\hat{\alpha}_{ak}$ is relatively minor. Equation (20) implies that the actuator loop becomes a low-pass filter with respect to the feedforward signal $\boldsymbol{I}_f$. The feedback signal $\boldsymbol{I}_m$ still passes the actuator loop without distortion. In such a case, the adaptive *corrective* control law (15)–(18) is correspondingly modified as

$$\begin{aligned} \boldsymbol{u} &= L_0\dot{\boldsymbol{I}}_m + R_0\boldsymbol{I}_d + K_{eo}\dot{\boldsymbol{q}}_d \\ &\quad - (\hat{\delta}_1\|\dot{\boldsymbol{I}}_m\| + \hat{\delta}_2\|\boldsymbol{I}_d\| + \hat{\delta}_3\|\dot{\boldsymbol{q}}_d\|)\operatorname{sgn}(\tilde{\boldsymbol{I}}) && (21) \\ \dot{\hat{\delta}}_1 &= \eta_1\|\dot{\boldsymbol{I}}_m\|\|\tilde{\boldsymbol{I}}\| && (22) \\ \dot{\hat{\delta}}_2 &= \eta_2\|\boldsymbol{I}_d\|\|\tilde{\boldsymbol{I}}\| && (23) \\ \dot{\hat{\delta}}_3 &= \eta_3\|\dot{\boldsymbol{q}}_d\|\|\tilde{\boldsymbol{I}}\|. && (24) \end{aligned}$$

The stability of the closed-loop system is therefore stated in the following theorem.

Theorem 2: If the estimated inertia parameters $\hat{\alpha}_{ak}$ are bounded, then in the closed-loop system the vectors $\tilde{\boldsymbol{q}}$ and $\tilde{\boldsymbol{I}}$ are both asymptotically stable, provided γ initially satisfies

$$\gamma\lambda_q > 3\|B_d\| + \mu_1 + 2\vartheta\left(\sup\|\dot{\boldsymbol{q}}_d\| + \sqrt{\frac{2}{\lambda_p}V}\right), \tag{25}$$

where λ_p and λ_q are defined in (12); V is a function defined in (33) while

$$\mu_1 \triangleq \frac{\beta_1^2}{4\lambda_r}, \quad \beta_1 = (3 + \zeta_1 + \|K_e\|) \quad \text{and}$$

$$\lambda_r \triangleq \inf \frac{\tilde{\boldsymbol{I}}^T R \tilde{\boldsymbol{I}}}{\|\tilde{\boldsymbol{I}}\|^2}.$$

Proof: See Appendix C.

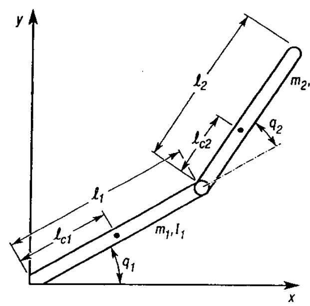

Fig. 2. Two-linkage manipulator.

Remark: The validity of the simplified algorithm depends on the boundedness of the estimated inertia parameters $\hat{\alpha}_{ak}$. Although the boundedness of $\hat{\alpha}_{ak}$ is verified in simulation results, the strict proof in theory remains an open question. However, as long as $\hat{\alpha}_{ak}$ is uniformly bounded, the stability of the closed-loop, using $\dot{\boldsymbol{I}}_m$ instead of $\dot{\boldsymbol{I}}_d$, can be guaranteed, and the simplified algorithm is of the same complexity as the algorithm by Slotine and Li [13].

IV. A Simulation Example

A. System Description

As an illustration, we will apply the adaptive algorithm (21)–(24) to a two-link robot arm with DC actuators proposed as a benchmark robotic system in [17] shown in Fig. 2. The robot model is described by (1), (2). A parameterization scheme for this robot is given in [8]

$$\begin{aligned} \alpha_1 &= m_2 l_1^2 + m_l l_1^2 + I_1 + I_2 + J_1 + I_l \\ \alpha_2 &= I_2 + J_2 + I_l \\ \alpha_3 &= I_2 + I_l \\ \alpha_4 &= m_2 l_1 (l_{c2} + l_2) + m_l l_1 (l_{cl} + l_2) \\ \alpha_5 &= m_2 l_1 + m_1 (l_1 + l_{c1}) + m_l l_1 \\ \alpha_6 &= m_2 (l_2 + l_{c2}) + m_l (l_2 + l_{cl}). \end{aligned} \tag{26}$$

where m_l is the mass of the end-effector and load, I_l is the inertia of the end-effector and load, l_{cl} is the mass m_l center of gravity coordinate, J_1, J_2 are the rotor inertias.

With this parameterization, $\Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)$ in (4) has components

$$\begin{aligned} &\phi_{11} = \ddot{q}_{d1} \quad \phi_{12} = 0 \quad \phi_{13} = \ddot{q}_{d2} \\ &\phi_{14} = \cos(q_2)(2\ddot{q}_{d1} + \ddot{q}_{d2}) - \sin(q_2)(\dot{q}_{d2}^2 + 2\dot{q}_{d1}\dot{q}_{d2}) \\ &\phi_{15} = g\cos(q_1) \quad \phi_{16} = g\cos(q_1 + q_2) \\ &\phi_{21} = 0 \quad \phi_{22} = \ddot{q}_{d2} \quad \phi_{23} = \ddot{q}_{d1} \\ &\phi_{24} = \cos(q_2)\ddot{q}_{d1} + \sin(q_2)\dot{q}_{d1}^2 \\ &\phi_{25} = 0 \quad \phi_{26} = g\cos(q_1 + q_2) \end{aligned} \tag{27}$$

The values of the manipulator and actuator parameters are given by [17] $l_1 = 0.45\,\mathrm{m}, m_1 = 100$ kg, $l_{c1} = 0.15\,\mathrm{m}, I_1 = 6.25\,\mathrm{kg\,m^2}, J_1 = 4.77\,\mathrm{kg\,m^2}, l_2 = 0.20\,\mathrm{m}, m_2 = 25\,\mathrm{kg}, l_{c2} = 0.10\,\mathrm{m}, I_2 = 0.61\,\mathrm{kg\,m^2}, J_2 = 3.58\,\mathrm{kg\,m^2}, m_l = 40\,\mathrm{kg}, l_{cl} = 0.20\,\mathrm{m}, I_l = 7.68\,\mathrm{kg\,m^2}, L_1 = 8 \times 10^{-5}$ Vs/A, $R_1 = 1.5\,\mathrm{Ohm}, K_{e1} = 25.05\,\mathrm{Vs}, K_{N1} = 25.05\,\mathrm{Vs}, L_2 = 8 \times 10^{-5}$ Vs/A, $R_2 = 1.5\,\mathrm{Ohm}, K_{e2} = 21.07\,\mathrm{Vs}, K_{N2} = 21.07\,\mathrm{Vs}$.

We also need to choose the nominal system parameters. Let the uncertainty of the inertia parameters be originated by the varying load m_l. The electrical parameters are assumed to have 50% of uncertainty. The nominal system parameters are given by $L_1 = 5 \times 10^{-5}$ Vs/A, $R_1 = 1.0\,\mathrm{Ohm}, K_{e1} = 16.53\,\mathrm{Vs}, L_2 = 5 \times 10^{-5}$ Vs/A, $R_2 = 1.0\,\mathrm{Ohm}, K_{e2} = 14.54\,\mathrm{Vs}, m_l = 20$ kg.

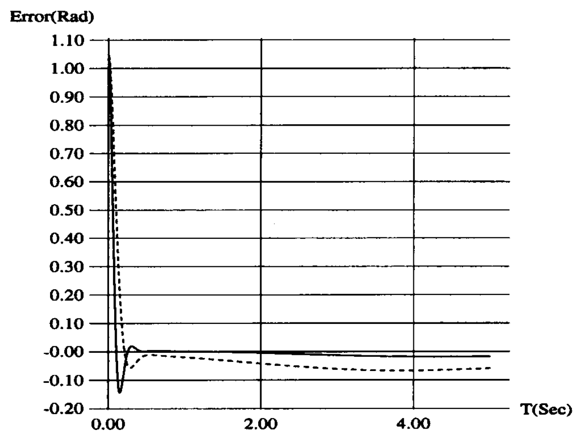

Fig. 3. Error comparison for joint 1.

The desired $\boldsymbol{I}_d$ is synthesized by (6) where $\kappa = 8, \gamma^2 = 10, \Gamma = 15I$, and $\sigma = 0.2$. The initial values of $\hat{\alpha}_{ak}$ are chosen as $\hat{\alpha}_{ak}(0) =$ [1.0657, 0.3575, 0.1888, 0.1051, 2.1869, 2.2911, 1.2297, 0.4125, 0.2179, 0.1213, 2.5234, 2.6434]^T. The controller is then synthesized by (15) where $\eta_1 = 1 \times 10^{-11}, \eta_2 = 1 \times 10^{-6}$, and $\eta_3 = 1 \times 10^{-6}$. The initial values of $\hat{\delta}_i$ are chosen as $\hat{\delta}_1(0) = 8 \times 10^{-5}, \hat{\delta}_1(0) = 1$, and $\hat{\delta}_1(0) = 10$.

B. Simulation Results

The control (15)–(18) is used to track the desired trajectories

$$q_{1d} = q_{2d} = -90^\circ + 52.5(1 - \cos 1.26t).$$

The initial displacements and velocities are chosen as $q_1(0) = -30^\circ, q_2(0) = -70^\circ, \dot{q}_1(0) = \dot{q}_2(0) = 0$. The proposed hybrid controller is compared with the adaptive controller (6) that neglects the motor dynamics. the results of the simulation are shown in Figs. 3–4 for two cases: the controller taking the motor dynamics into consideration (solid curve) and the control neglecting the motor dynamics (dotted curve). Fig. 3 shows the trajectory tracking error of joint 1. Fig. 4 shows the trajectory tracking error of joint 2. It is confirmed that, compared with a controller based on the manipulator dynamics only, the controller based on manipulator dynamics as well as the motor dynamics obviously provides a better tracking performance.

It is noted, however, that the use of the controller that takes the actuator dynamics into consideration will increase the computational load. The numbers of computation involved to update the control law is about three times larger as compared with the controller (6) that neglects the motor dynamics. One way to reduce the computation load is to use the simplified algorithm (21). In this case, the real-time implementation aspects are similar to [13] and may be referred to [13]. Fig. 5 shows the tracking error of joint 1. Fig. 6 shows the tracking error of joint 2. The tracking errors have very similar transient patterns as compared to the control (15). These results coincides with the analysis in Section III. It should be mentioned that in the simulation the estimated inertia parameters are converged to bounded values.

V. Conclusion

In this paper, a hybrid adaptive/robust control law, based on the cascade control strategy, has been derived that incorporates

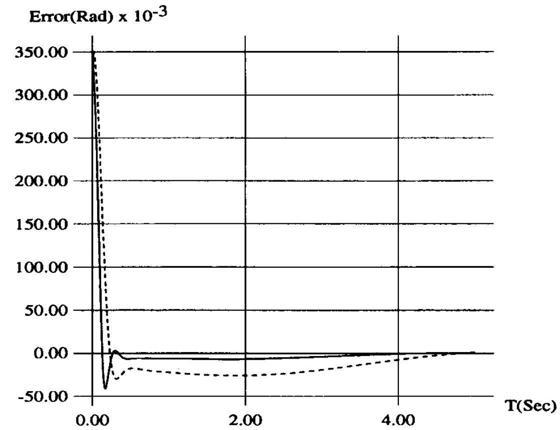

Fig. 4. Error comparison for joint 2.

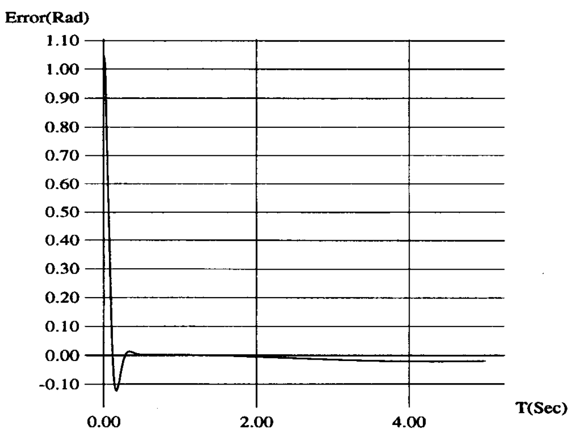

Fig. 5. Tracking error of joint 1 with simplified algorithm.

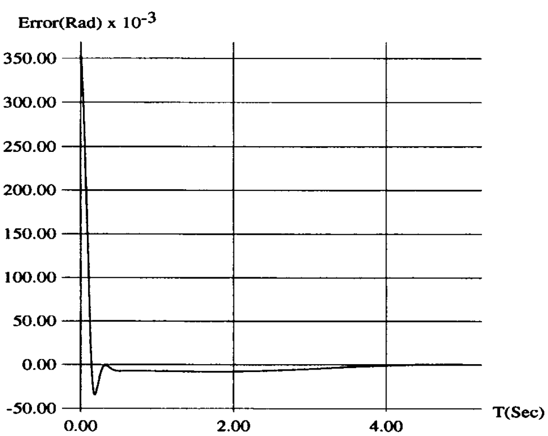

Fig. 6. Tracking of error of joint 2 with simplified algorithm.

the robot manipulator dynamics as well as the dynamics of the robot joint motors, in the case of arbitrary uncertain mechanical and electrical parameters of the robotic system. The control law requires the measurement of only joint positions, velocities and motor armature currents. Asymptotical stability of the closed-loop system is established in the Lyapunov sense. Simulations were performed with a two-link example, and simulation results verify the improvement in performance which was expected to be obtained by including the actuator dynamics in the control design. Compared to the controllers designed in torque level [1], [11], the present scheme needs additionally the measurement of the motor armature current and a somewhat more complicated control law, which, however, should be viewed together with the benefit of improving tracking performance.

Appendix A
Proof of Lemma 1

Consider a Lyapunov function candidate

$$L_a \triangleq L_d + \frac{1}{2\sigma}\tilde{\alpha}_{ak}^T\tilde{\alpha}_{ak}. \tag{28}$$

where $L_d \triangleq (1/2)\boldsymbol{x}^T P\boldsymbol{x}$. Its time derivative is evaluated along the trajectory of (11) as

$$\begin{aligned}\dot{L}_a = &-\boldsymbol{x}^T Q\boldsymbol{x} + \boldsymbol{x}^T PC(-K_N^{-1}B(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}}\\ &- K_N^{-1}B_d\dot{\tilde{\boldsymbol{q}}} + \Phi_a\tilde{\alpha}_{ak}) + \frac{1}{2}\boldsymbol{x}^T\dot{P}\boldsymbol{x} + \frac{1}{\sigma}\dot{\tilde{\alpha}}_{ak}^T\tilde{\alpha}_{ak}.\end{aligned} \tag{29}$$

When $\gamma \geq \max\{1,\kappa\}$, one can write

$$\begin{aligned}-\boldsymbol{x}^T PCK_N^{-1}B_d\dot{\tilde{\boldsymbol{q}}} &= -\left(\dot{\tilde{\boldsymbol{q}}} - \frac{1}{\gamma}\boldsymbol{w} + \frac{\kappa}{\gamma}\tilde{\boldsymbol{q}}\right)^T B_d\dot{\tilde{\boldsymbol{q}}}\\ &\leq 3\|B_d\|\|\boldsymbol{x}\|^2\end{aligned} \tag{30}$$

and

$$\begin{aligned}&\frac{1}{2}\boldsymbol{x}^T\dot{P}\boldsymbol{x} - \boldsymbol{x}^T PCK_N^{-1}B(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}}\\ &\quad= \frac{1}{\gamma}(\kappa\tilde{\boldsymbol{q}} - \boldsymbol{w})^T[\dot{D} - B(\boldsymbol{q},\dot{\boldsymbol{q}})]\dot{\tilde{\boldsymbol{q}}}\\ &\quad\leq 2\vartheta\|\dot{\boldsymbol{q}}\|\|\boldsymbol{x}\|^2,\end{aligned} \tag{31}$$

where $\vartheta\|\dot{\boldsymbol{q}}\| = \|\dot{D} - B\|$ and identity $\dot{\tilde{\boldsymbol{q}}}^T[(1/2)\dot{D} - B(\boldsymbol{q},\dot{\boldsymbol{q}})]\dot{\tilde{\boldsymbol{q}}} = 0$ has been used to derive (31). Substituting (12), (30) and (31) into (29), one obtains

$$\begin{aligned}\dot{L}_a &\leq -(\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\dot{\boldsymbol{q}}\|)\|\boldsymbol{x}\|^2\\ &\quad+ \left(\boldsymbol{z}^T\Phi_a + \frac{1}{\sigma}\dot{\tilde{\alpha}}_{ak}^T\right)\tilde{\alpha}_{ak}\\ &= -f(\|\dot{\boldsymbol{q}}\|)\|\boldsymbol{x}\|^2\end{aligned} \tag{32}$$

where $f(\|\dot{\boldsymbol{q}}\|) \triangleq (\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\dot{\boldsymbol{q}}\|)$ and identity $\boldsymbol{x}^T PC\Phi_a\tilde{\alpha}_{ak} = \boldsymbol{z}^T\Phi_a\tilde{\alpha}_{ak}$ and the (8) have been used. The right side of (32) is negative if $f(\|\dot{\boldsymbol{q}}\|) > 0$, which is true if (13) is satisfied.

When $\gamma\lambda_q$ is sufficiently large such that (13) is satisfied for $t = t_0$, then $L_a(t_0) < 0$. This indicates a decreasing L_a and continuous satisfaction of (13) for $t = t_0 + \delta t$, where δt is an infinitesimal of time. By induction with respect to t, $L_a(t)$ will keep on decreasing until $\|\boldsymbol{x}\| = 0$.

Appendix B
Proof of Theorem 1

The closed-loop stability is related to a Lyapunov function candidate

$$V(t) = L_a(t) + L_i(t) \tag{33}$$

where $L_a(t)$ is defined in (28) and

$$L_i(t) \triangleq \tfrac{1}{2}\tilde{\boldsymbol{I}}^T L\tilde{\boldsymbol{I}} + \tfrac{1}{2}\sum_{i=1}^{3}(\delta_i - \hat{\delta}_i)^2/\eta_i.$$

where δ_i is defined in (14) and $\hat{\delta}_i$ is its estimate.

The time derivative of $L_a(t)$ should not be bounded from above by (32) since $\tilde{I}$ is not necessarily an all-zero vector. Instead, an additional term $x^T PC\tilde{I}$ must be added to the right side of (32) to establish an upper bound for $\dot{L}_a$ when $\tilde{I} \neq 0$. as a result, one has to write

$$\dot{L}_a \leq -f(\|\dot{q}\|)\|x\|^2 + x^T PC\tilde{I}. \quad (34)$$

When $\gamma \geq \max\{1, \kappa\}$, one can write

$$x^T PC\tilde{I} = \left(\dot{\tilde{q}} - \frac{1}{\gamma}w + \frac{\kappa}{\gamma}\tilde{q}\right)^T \tilde{I} \leq 3\|x\|\|\tilde{I}\|. \quad (35)$$

Consequently,

$$\dot{L}_a \leq -f(\|\dot{q}\|)\|x\|^2 + 3\|x\|\|\tilde{I}\|. \quad (36)$$

The time derivative of $L_i(t)$ is evaluated along the trajectory (2) as

$$\dot{L}_i = -\tilde{I}^T[L\dot{I}_d + R\tilde{I} + K_e\dot{\tilde{q}} - u + RI_d + K_e\dot{q}_d] + \sum_{i=1}^{3}(\delta_i - \hat{\delta}_i)(-\dot{\hat{\delta}}_i)/\eta_i. \quad (37)$$

Substituting u in (37) by the control law (15), one obtains

$$\begin{aligned}\dot{L}_i \leq & -\tilde{I}^T R\tilde{I} - \tilde{I}^T K_e\dot{\tilde{q}} + (\delta_1\|\dot{I}_d\|\|\tilde{I}\| \\ & + \delta_2\|I_d\|\|\tilde{I}\| + \delta_3\|\dot{q}_d\|\|\tilde{I}\|) \\ & - (\hat{\delta}_1\|\dot{I}_d\|\|\tilde{I}\| + \hat{\delta}_2\|I_d\|\tilde{I}\| + \hat{\delta}_3\|\dot{q}_d\|\|\tilde{I}\|) \\ & + \sum_{i=1}^{3}(\delta_i - \hat{\delta}_i)(-\dot{\hat{\delta}}_i)/\eta_i \\ = & -\tilde{I}^T R\tilde{I} - \tilde{I}^T K_e\dot{\tilde{q}} \leq -\tilde{I}^T R\tilde{I} + \alpha_k\|x\|\|\tilde{I}\|. \quad (38)\end{aligned}$$

where $\alpha_k \triangleq \|K_e\|$.

Based on (36) and (38), $\dot{V}$ can be expressed as

$$\begin{aligned}\dot{V} & \leq -f(\|\dot{q}\|)\|x\|^2 + \beta\|x\|\|\tilde{I}\| - \tilde{I}^T R\tilde{I} \\ & \leq -(f(\|\dot{q}\|) - \mu)\|x\|^2 - \lambda_r(\|\tilde{I}\| - \nu\|x\|)^2, \quad (39)\end{aligned}$$

where

$$\beta \triangleq (3 + \alpha_k), \quad \mu \triangleq \frac{\beta^2}{4\lambda_r}, \quad \nu \triangleq \frac{\beta}{2\lambda_r} \quad \text{and}$$

$$\lambda_r \triangleq \inf \frac{\tilde{I}^T R\tilde{I}}{\|\tilde{I}\|^2}.$$

The right side of (39) is negative if $(f(\|\dot{q}\|) - \mu) > 0$, which is true if (19) is satisfied. The region of the attraction for V is given by (19).□

APPENDIX C
PROOF OF THEOREM 2

In this case the Lyapunov function candidate (33) is modified as

$$V(t) = L_a(t) + \overline{L}_i(t) \quad (40)$$

where $L_a(t)$ is defined in (28) and

$$\overline{L}_i(t) \triangleq \tfrac{1}{2}\tilde{I}^T L\tilde{I} + \tfrac{1}{2}\sum_{i=1}^{3}(\overline{\delta}_i - \hat{\delta}_i)^2/\eta_i.$$

where $\overline{\delta}_1 = \delta_1, \overline{\delta}_2 = \delta_2$, and $\overline{\delta}_3 = (\delta_3 + \zeta_2), \delta_i$ are defined in (14), ζ_2 is defined in (43), and $\hat{\delta}_i$ are the estimates of $\overline{\delta}_i$.

The time derivative of the first term $L_a(t)$ in (40) is unchanged. Due to the change of the control u, the derivative of $\overline{L}_i(t)$ becomes

$$\dot{\overline{L}}_i = -\tilde{I}^T[L(\dot{I}_m + \dot{I}_f) + R\tilde{I} + K_e\dot{\tilde{q}} - u + RI_d + K_e\dot{q}_d] + \sum_{i=1}^{3}(\overline{\delta}_i - \hat{\delta}_i)(-\dot{\hat{\delta}}_i)/\eta_i. \quad (41)$$

When $\gamma \geq \max\{1, \kappa\}$, one can write

$$\begin{aligned}-\tilde{I}^T L\dot{I}_f & = -\tilde{I}^T L(\dot{\Phi}_a\hat{\alpha}_{ak} + \Phi_a\dot{\hat{\alpha}}_{ak}) \\ & \leq \alpha_l\|\tilde{I}\|(\|\dot{\Phi}_a\|\|\hat{\alpha}_{ak}\| + \|\Phi_a\|\|\dot{\hat{\alpha}}_{ak}\|) \\ & \leq \alpha_l\|\tilde{I}\|(\|\dot{\Phi}_a\|\|\hat{\alpha}_{ak}\| + 3\sigma\|\Phi_a\|^2\|\|x\|) \quad (42)\end{aligned}$$

where $\alpha_l \triangleq \|L\|$ and the (8) and (9) have been used. Since $\dot{q}_d, \ddot{q}_d$ and $q_d^{(3)}$ are uniformly bounded, one can write

$$\|\Phi_a\| \leq \rho; \quad \|\dot{\Phi}_a\| \leq \varrho\|\dot{q}\|$$

where ρ and ϱ are constants. Thus, the (42) becomes

$$\begin{aligned}-\tilde{I}^T L\dot{I}_f & \leq \alpha_l\varrho\|\tilde{I}\|\|\dot{q}\|\|\hat{\alpha}_{ak}\| + 3\alpha_l\sigma\rho^2\|\tilde{I}\|\|x\| \\ & \leq \alpha_l\varrho\|\tilde{I}\|(\|\dot{\tilde{q}}\|) + \|\dot{q}_d\|)\|\hat{\alpha}_{ak}\| + 3\alpha_l\sigma\rho^2\|\tilde{I}\|\|x\| \\ & \leq \zeta_1\|\tilde{I}\|\|x\| + \zeta_2\|\dot{q}_d\|\|\tilde{I}\| \quad (43)\end{aligned}$$

where $\zeta_1 \triangleq (\alpha_l\varrho\|\hat{\alpha}_{ak}\| + 3\alpha_l\sigma\rho^2)$ and $\zeta_2 \triangleq \alpha_l\varrho\|\hat{\alpha}_{ak}\|$.

Substituting u in (41) by the control law (21) and noticing (14) and (43), one obtains

$$\begin{aligned}\dot{L}_i \leq & -\tilde{I}^T R\tilde{I} - \tilde{I}^T K_e\dot{\tilde{q}} - \tilde{I}^T L\dot{I}_f \\ & + (\delta_1\|\dot{I}_m\|\|\tilde{I}\| + \delta_2\|I_d\|\|\tilde{I}\| + \delta_3\|\dot{q}_d\|\|\tilde{I}\|) \\ & - (\hat{\delta}_1\|\dot{I}_m\|\|\tilde{I}\| + \hat{\delta}_2\|I_d\|\tilde{I}\| + \hat{\delta}_3\|\dot{q}_d\|\|\tilde{I}\|) \\ & + \sum_{i=1}^{3}(\overline{\delta}_i - \hat{\delta}_i)(-\dot{\hat{\delta}}_i)/\eta_i \\ \leq & -\tilde{I}^T R\tilde{I} - \tilde{I}^T K_e\dot{\tilde{q}} + \zeta_1\|\tilde{I}\|\|x\| \\ \leq & -\tilde{I}^T R\tilde{I} + (\zeta_1 + \alpha_k)\|x\|\|\tilde{I}\| \quad (44)\end{aligned}$$

Based on (36) and (44), $\dot{V}$ can be expressed as

$$\begin{aligned}\dot{V} & \leq -f(\|\dot{q}\|)\|x\|^2 + \beta_1\|x\|\|\tilde{I}\| - \tilde{I}^T R\tilde{I} \\ & \leq -(f(\|\dot{q}\|) - \mu_1)\|x\|^2 - \lambda_r(\|\tilde{I}\| - \nu_1\|x\|)^2, \quad (45)\end{aligned}$$

where

$$\beta_1 \triangleq (3 + \zeta_1 + \alpha_k), \quad \mu_1 \triangleq \frac{\beta_1^2}{4\lambda_r}, \quad \nu_1 \triangleq \frac{\beta_1}{2\lambda_r} \quad \text{and}$$

$$\lambda_r \triangleq \inf \frac{\tilde{I}^T R\tilde{I}}{\|\tilde{I}\|^2}.$$

The right side of (45) is negative if $(f(\|\dot{q}\|) - \mu_1) > 0$, which is true if (25) is satisfied. □

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Learning Control for Robot Tasks under Geometric Endpoint Constraints

Tomohide Naniwa, *Member, IEEE*, and Suguru Arimoto, *Fellow, IEEE*

***Abstract*—A learning control scheme for a class of robot manipulators whose endpoint is moving under geometrical constraints on a surface is proposed. In this scheme, the input torque command is composed of two different signals updated separately at every trial by different ways. One is updated by the angular velocity error vector which is projected to the tangent plane of the constraint surface in joint space. The other is updated by the magnitude of contact force error at the manipulator endpoint.**

Not only the uniform boundedness of position and velocity trajectory errors but also the uniform convergence of position and velocity trajectories to their desired ones with repeating practices are proved theoretically. In addition, it is shown that the contact force itself converges to the desired one in the sense of L^2-norm with repeating practices. Computer simulation results by using a 3 DOF manipulator are presented to demonstrate the effectiveness of the proposed method and to examine the speed of convergence of force trajectories besides position and velocity trajectories.

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The authors are with the Department of Mathematical Engineering and Information Physics, Faculty of Engineering, University of Tokyo, Bunkyo-ku, Tokyo, 113 Japan.
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I. Introduction

Skillfulness of motions done by humans has been acquired through repeated practices. Cleaning a window, writing with a pen, and eating with knife and fork are typical tasks that humans do in their everyday life. Humans perform all those tasks skillfully and almost unconsciously, but they had to learn much from experience through practices. In contrast, can mechanical robots acquire such skillfulness of motions through repeated practices? Motivated by this observation, "Learning Control" that is a new approach for the control problem of skill refinement through practices has been studied extensively in the previous literatures [1]–[4].

Learning control is crystallized into a simple recursive form of learning law, in which the next actuator input torque is composed of the previous input torque plus a modification term that refers to previous angular velocity or/and acceleration errors. Therefore, learning control laws need not use the knowledge of dynamics of the manipulator.

Many of previous studies of learning control [1]–[4] treat the case that an endpoint of the manipulator can move freely without any constraint in work space. However, there is a large class of tasks in which the endpoint of the manipulator must contact with the environment or other objects and sometime must move in touch with it. In such cases, dynamics of the manipulator includes two terms of contact force and friction force arising at the contact point and at the same time must satisfy an algebraic constraint equation. In addition, the manipulator must be controlled to achieve not only the desired position trajectory but also the desired contact force trajectory.

A learning control problem for manipulators whose endpoint is moving in touch with the environment was first treated by Kawamura *et al.* [5], where the contact surface is assumed to be stiff but not rigid and hence it is assumed to move in one direction. In their treatment the contact force error is assumed to be proportional to the displacements of the surface. Very recently Jeon and Tomizuka proposed a different but similar approach called "repetitive learning control" of manipulators for such a stiff contact case [6] where the task must be periodic in the nature of repetitive control. In all these methods, the convergence of contact force exerted at the endpoint is proved in terms of the convergence of displacements of the surface. A general case of rigid contact was first treated by Aicardi *et al.* [7]. They introduced "mixed" dynamics of the manipulator described in the new mixed coordinates which is composed of a part of the manipulator joint angle coordinates and the Cartesian coordinates at the endpoint. By using an approximated "mixed" dynamics model, the feed-forward control input is calculated, which includes an acceleration term. Therefore, the computational cost of calculation of the control inputs is not trivial. They showed theoretically the convergence of position and force trajectories by assuming high PD feedback gains so that in all trials those trajectories must remain in a neighborhood of the desired ones.

Another learning control method was proposed by Lucibello [8] which constructs the state space from the contact force and the reduced coordinates of joint angle coordinates that are derived by the implicit function theorem. This method is based on the feedback linearization. Therefore, it needs to use the dynamics model of the manipulator.

A simple recursive form of learning control law is proposed for a general rigid contact case in our recent paper [9], which is based on only a modification of angular velocity errors. This scheme needs accurate values of the inertial matrix of manipulator