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# BACKSTEPPING-BASED HYBRID ADAPTIVE CONTROL OF ROBOT MANIPULATORS INCORPORATING ACTUATOR DYNAMICS

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## SUMMARY

By using the integrator backstepping technique, the control of rigid link, electrically driven robot manipulators is addressed in the presence of arbitrary uncertain manipulator inertia parameters and actuator parameters. The control scheme developed is computationally simple owing to the avoidance of the derivative computation of the regressor matrix. Semiglobal asymptotic stability of the controller is established in the Lyapunov sense. Simulation results are included to demonstrate the tracking performance.


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## 1. INTRODUCTION

Recently actuator dynamics have been explicitly included in control schemes. These dynamics become extremely important during fast robot motion and highly varying loads. However, as demonstrated by Good *et al.*,[1] the inclusion of actuators in the dynamic equations complicates both the controller structure and its stability analysis. This is because the inclusion of the robot actuator dynamics in the robot dynamic equations makes the latter a system of third-order differential equations.[2]

The study of the control of rigid robots including actuators has been described e.g. in References 2–7. The early works[2,3,7] pioneered the development of control methods. However, their design procedures are based on full knowledge of the robotic dynamics. If there are uncertainties in the system dynamics, controllers designed in this way may give degraded performance and may incur instability. The schemes given in References 5 and 8 only deal with uncertainty in the manipulator and require full knowledge of the actuator parameters. To deal

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with uncertainties in the combined dynamics, using the integrator backstepping technique,[9] some promising robust schemes were recently proposed in References 4, 6, 10 and 11.

This paper attacks the same problem as References 4, 6, and 11, i.e. posed in the tracking control of rigid link, electrically driven robots with both manipulator and actuator uncertainties. Using the integrator backstepping technique,[9] a hybrid adaptive controller (i.e. adaptive and robust adaptive) is proposed. However, it should be noted that our scheme is not merely a simple extension of that in References 9 and 12, since link acceleration measures are not allowed and the derivative calculation of some complexity functions (the regressor matrix) is avoided. Compared with existing schemes, the proposed controller has the following features: it does not require the derivative computation of complexity functions or upper bounds of the derivative of complexity functions. Semiglobal asymptotic stability of the controller is established in the Lyapunov sense.

The arrangement of this paper is as follows. In Section 2 the robot dynamics including actuators are expressed in the form of two cascaded loops: the actuator loop and the manipulator loop. An *embedded* force is introduced as a synthesized input signal intended for the manipulator loop. Using the *embedded* control signals, a control law is then synthesized for the usually neglected electrical actuator loop. Semiglobal asymptotic stability of the controller is established in the Lyapunov sense. Simulation results are discussed in Section 3 and conclusions are given in Section 4.

## 2. DERIVATION OF THE CONTROL LAW

Consider an $n$-link manipulator with revolute joints driven by armature-controlled DC motors with voltages being inputs to amplifiers. As in References 2, 4 and 6, the dynamics are described by

$$(D(\mathbf{q}) + J)\ddot{\mathbf{q}} + B(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) = K_N\mathbf{I} \tag{1}$$

$$L\dot{\mathbf{I}} + R\mathbf{I} + K_e\dot{\mathbf{q}} = u \tag{2}$$

where $\mathbf{q} \in \mathbb{R}^n$ is the vector of joint positions, $\mathbf{I} \in \mathbb{R}^n$ is the vector of armature currents and $\mathbf{u} \in \mathbb{R}^n$ is the vector of armature voltages; $D(\mathbf{q})$ is the manipulator mass matrix, which is a symmetric positive definite matrix; $B(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ represents the centripetal and Coriolis forces; $G(\mathbf{q})$ denotes the gravitational force; $J$ is the actuator inertia matrix; $L$ represents the actuator inductance matrix; $R$ is the actuator resistance matrix, $K_e$ is the matrix characterizing the voltage constant of the actuator and $K_N$ is the matrix which characterizes the electromechanical conversion between current and torque. While $D(\mathbf{q})$, $B(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ and $G(\mathbf{q})$ are non-linear functions, $J$, $L$, $R$, $K_e$ and $K_N$ are positive definite constant diagonal matrices. We note only that the matrix $\dot{D} - 2B$ is a skew-symmetric matrix.

It is assumed that $\dot{\mathbf{q}}$, $\mathbf{q}$ and $\mathbf{I}$ are measurable and the exact values of the robotic functions $D(\mathbf{q})$, $B(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ and $G(\mathbf{q})$ and actuator dynamic coefficient matrices $J$, $L$, $R$, $K_e$ and $K_N$ are not available. the considered adaptive controller design problem is as follows. For any given desired bounded trajectories $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$, $\ddot{\mathbf{q}}_d$ and $\mathbf{q}_d^{(3)}$, with some or all of the manipulator parameters unknown, derive a controller for the actuator voltages $\mathbf{u}$ such that the manipulator position vector $\mathbf{q}(t)$ tracks $\mathbf{q}_d(t)$.

The dynamic model (1), (2) actually consists of two cascaded loops. Unlike dynamic models of robots assuming that the joint torque can be commanded directly, the torque $K_N\mathbf{I}$ in (1) cannot be synthesized directly. Instead, it is an output of the actuator dynamics. In accordance with the backstepping control strategy described in References 4, 6 and 13, the design procedure is organized as a two-step process. Firstly, the vector $\mathbf{I}$ is regarded as a control variable for subsystem (1) and an *embedded* control input $\mathbf{I}_d$ is designed so that the tracking goal may be achieved. Secondly, $\mathbf{u}$ is designed such that $\mathbf{I}$ tracks $\mathbf{I}_d$. In turn, this allows $\mathbf{q}(t)$ to track $\mathbf{q}_d(t)$. In this paper, (1) is called the *manipulator loop* and (2) the *actuator loop.*

*2.1. Adaptive control for the manipulator loop*

Using the *embedded* armature current vector $\mathbf{I}_d$, the model (1) can be rewritten as

$$(D(\mathbf{q}) + J)\ddot{\mathbf{q}} + B(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) = K_N\mathbf{I}_d + K_N\tilde{\mathbf{I}} \tag{3}$$

where $\tilde{\mathbf{I}} \triangleq \mathbf{I} - \mathbf{I}_d$ represents a current perturbation to the rigid link dynamics. The system (3) can be viewed as a rigid model system with an input disturbance $K_N\tilde{\mathbf{I}}$, controlled by $K_N\mathbf{I}_d$. The synthesis of $K_N\mathbf{I}_d$ may follow any available design procedure developed at the torque input level.

However, the direct application of design procedures developed at the torque input level to design $I_d$ is impaired by the assumption that the electromechanical conversion matrix $K_N$ is not exactly available, so $\mathbf{I}_d$ cannot be calculated from $K_N\mathbf{I}_d$. Therefore one needs a modified scheme to directly generate the signal $\mathbf{I}_d$.

In order to solve this problem, based on the parametrization technique in Reference 6, the non-linear terms $D$, $B$ and $G$ in (1) can be expressed as

$$(D(\mathbf{q}) + J)\ddot{\mathbf{q}}_d + B(\mathbf{q}, \dot{\mathbf{q}}_d)\dot{\mathbf{q}}_d + G(\mathbf{q}) = \Phi_a(\mathbf{q}, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)\alpha_a \tag{4}$$

where the term $\Phi_a(\mathbf{q}, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d) \in \mathbb{R}^{n\times(n\times m)}$ is the augmented regressor matrix independent of the dynamic parameters; the term $\alpha_a^T \triangleq [\alpha^T \ \alpha^T \ \cdots \ \alpha^T]$ is a corresponding augmented inertia parameter vector, wherein $\alpha \in \mathbb{R}^m$ is a constant vector of manipulator inertia parameters. Then

$$K_N^{-1}\Phi_a\alpha_a = \Phi_a K_{Na}^{-1}\alpha_a = \Phi_a\alpha_{ak} \tag{5}$$

where $K_{Na} \triangleq \text{diag}[k_{Ni}I_m]$ and $\alpha_{ak}^T \triangleq [k_{N1}^{-1}\alpha^T, k_{N2}^{-1}\alpha^T, \ldots, k_{Nn}^{-1}\alpha^T]$.

We suppose only that the parameter vector $\alpha_{ak}$ is 'uncertain'. Following the results of Reference 13, the desired $\mathbf{I}_d$ is then synthesized by

$$\mathbf{I}_d = \Phi_a(\mathbf{q}, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)\hat{\alpha}_{ak} - \gamma^2\Gamma(\mathbf{w} + \kappa\tilde{\mathbf{q}}) \tag{6}$$

where $\tilde{\mathbf{q}} \triangleq \mathbf{q} - \mathbf{q}_d$ is the joint tracking error, $\Gamma$ is an arbitrary positive definite constant diagonal matrix, $\gamma$ and $\kappa$ are positive constants and $\mathbf{w}$ is an intermediate vector synthesized by

$$\dot{\mathbf{w}} = -2\gamma\mathbf{w} + \gamma^2\dot{\tilde{\mathbf{q}}} \tag{7}$$

The adaptive law for adjusting $\hat{\alpha}_{ak}$ is given by

$$\dot{\hat{\alpha}}_{ak} = \dot{\tilde{\alpha}}_{ak} = -\sigma\Phi_a^T\mathbf{z} \tag{8}$$

$$\mathbf{z} \triangleq \dot{\tilde{\mathbf{q}}} - \frac{1}{\gamma}\mathbf{w} + \frac{\kappa}{\gamma}\tilde{\mathbf{q}} \tag{9}$$

where $\tilde{\alpha}_{ak} \triangleq \hat{\alpha}_{ak} - \alpha_{ak}$ denotes the parameter error vector and $\sigma$ is a positive constant.

It should be mentioned that $\mathbf{I}_d$ gives by the control law (6), (7) and adaptive law (8), (9) does not involve the velocity feedback $\dot{\mathbf{q}}$. This fact will be used later to prove that the controller of the overall system will depend only on the measurements of $\mathbf{I}$, $\mathbf{q}$ and $\dot{\mathbf{q}}$.

Substituting (6) into (3), one obtains the joint position error equations

$$K_N^{-1}(D(\mathbf{q}) + J)\ddot{\tilde{\mathbf{q}}} = -\gamma^2\Gamma\mathbf{w} - \kappa\gamma^2\Gamma\tilde{\mathbf{q}} + \tilde{\mathbf{I}} - K_N^{-1}B(\mathbf{q}, \dot{\mathbf{q}})\dot{\tilde{\mathbf{q}}} - K_N^{-1}B_d\dot{\tilde{\mathbf{q}}} + \Phi_a\tilde{\alpha}_{ak} \tag{10}$$

 

where $B_d \dot{\tilde{\mathbf{q}}} \triangleq B(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_d - B(\mathbf{q}, \dot{\mathbf{q}}_d)\dot{\mathbf{q}}_d$. Using the fact that $B(\mathbf{q}, \dot{\mathbf{q}})$ always satisfies $\| B(\mathbf{q}, \dot{\mathbf{q}}) \| \leqslant k_b \| \dot{\mathbf{q}} \|$ for all $(\mathbf{q}, \dot{\mathbf{q}})$, where $k_b$ is a constant,[14] it can be shown that $\| B_d \|$ is uniformly bounded when $\dot{\mathbf{q}}_d$ is uniformly bounded.

Introducing a state vector $\mathbf{x}^T \triangleq [\dot{\tilde{\mathbf{q}}}^T, \mathbf{w}^T, \tilde{\mathbf{q}}^T]$, the dynamic equation (10) can be expressed in state space as

$$\dot{\mathbf{x}} = -Ax + C(\tilde{\mathbf{I}} - K_N^{-1} B(\mathbf{q}, \dot{\mathbf{q}})\dot{\tilde{\mathbf{q}}} - K_N^{-1} B_d \dot{\tilde{\mathbf{q}}} + \Phi_a \tilde{\alpha}_{ak}) \tag{11}$$

where (7) is incorporated to obtain

$$A \triangleq \begin{bmatrix} 0 & \gamma^2 (D+J)^{-1} K_N \Gamma & \kappa\gamma^2 (D+J)^{-1} K_N \Gamma \\ -\gamma^2 I & 2\gamma I & 0 \\ -I & 0 & 0 \end{bmatrix}$$

$$C \triangleq \begin{bmatrix} (D+J)^{-1} K_N \\ 0 \\ 0 \end{bmatrix}$$

An important stage of the design procedure is to choose a pair of positive definite matrices $P$ and $Q$ such that $\frac{1}{2}(PA + A^T P) = Q$. One possible choice is given by

$$P \triangleq \begin{bmatrix} (D+J) & (-1/\gamma)(D+J) & (\kappa/\gamma)(D+J) \\ (-1/\gamma)(D+J) & K_N \Gamma & 0 \\ (\kappa/\gamma)(D+J) & 0 & \kappa\gamma^2 K_N \Gamma \end{bmatrix}$$

and $Q \triangleq \gamma Q_1$, where

$$Q_1 \triangleq \begin{bmatrix} (1 - \kappa/\gamma^2)(D+J) & -(D+J)/\gamma & 0 \\ -(D+J)/\gamma & K_N \Gamma & 0 \\ 0 & 0 & \kappa^2 K_N \Gamma \end{bmatrix}$$

Since the eigenvalues of $D$ are uniformly bounded for all $\mathbf{q}$, by choosing a sufficiently large $\gamma$, one can make $P$, $Q_1$ and therefore $Q$ positive definite. Thus we have

$$\gamma \lambda_q \| \mathbf{x} \|^2 \leqslant \mathbf{x}^T Q \mathbf{x} \tag{12}$$

where $\lambda_q$ denotes the smallest eigenvalue of the matrix $Q_1$.

Before the introduction of the control law of the actuator loop which compensates the disturbance $K_N \tilde{\mathbf{I}}$, it is helpful to study the closed-loop system stability of the manipulator loop when $\tilde{\mathbf{I}}$ is zero. The closed-loop system is described by (11) and (8). Its asymptotic stability is established by the following lemma.

*Lemma 1*

In the closed-loop system described by (11) and (8), all signals are bounded and $\lim_{t\to\infty} \tilde{\mathbf{q}} = 0$ provided that $\tilde{\mathbf{I}} = 0$ and $\gamma$ initially satisfies

$$\gamma \lambda_q > 3 \| B_d \| + 2\vartheta \| \mathbf{q}_d \| + 2\vartheta \sqrt{\left(\frac{\lambda_2}{\lambda_1}\right)} \| \mathbf{x}_\alpha(0) \| \tag{13}$$

where $\lambda_q$ is defined in (12), $\lambda_1$ and $\lambda_2$ are defined in (25) and $\mathbf{x}_\alpha^T = [x^T \ \ \tilde{\alpha}_{ak}^T]$.

*Proof.* See Appendix I.

 

*2.2. Hybrid adaptive control for the cascade control system*

We can now use (6) to design a control law at the voltage input $\mathbf{u}$ which forces $\tilde{\mathbf{I}}$ to zero. However, as shown in Reference 11, using the backstepping technique,[9,12] we are required to calculate

$$\dot{\mathbf{I}}_d = (d/dt)(\Phi_a(\mathbf{q}, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)\hat{\alpha}_{ak}) - \gamma^2\Gamma(\dot{\mathbf{w}} + \kappa\dot{\tilde{\mathbf{q}}})$$

where $(d/dt)(\Phi_a\hat{\alpha}_{ak}) = \dot{\Phi}_a\hat{\alpha}_{ak} + \dot{\Phi}_a\dot{\hat{\alpha}}_{ak}$. The computation of $\dot{\Phi}_a$ may be challenging. There seems to be no recursive way to compute $\dot{\Phi}_a$ for a general $n$-link manipulator in the literature. If such an algorithm were developed, it might be computationally expensive to update $\dot{\Phi}_a$. In order to avoid the intensive computation of $\dot{\Phi}_a$, as would be clear in the subsequent development, we can simply substitute

$$\dot{\mathbf{I}}_m \triangleq -\gamma^2\Gamma(\dot{\mathbf{w}} + \kappa\dot{\tilde{\mathbf{q}}}) \tag{14}$$

for $\dot{\mathbf{I}}_d$, since the feedback signal $\mathbf{I}_m = -\gamma^2\Gamma(\mathbf{w} + \kappa\tilde{\mathbf{q}})$ plays a vital role in the stability of the closed-loop systems, whereas the effect of the feedforward signal $\mathbf{I}_f \triangleq \Phi_a(\mathbf{q}, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)\hat{\alpha}_{ak}$ is relatively minor and can be compensated in the control law. Equation (14) implies that the actuator loop becomes a lowpass filter with respect to the feedforward signal $\mathbf{I}_f$. The feedback signal $\mathbf{I}_m$ still passes the actuator loop without distortion.

In the following development we assume that the electrical parameter $K_N$, $L$, $R$ and $K_e$ are all of uncertain values. However, there exist $L_0$, $R_0$ and $K_{e0}$, all known, such that

$$\| L - L_0 \| \leqslant \delta_1; \qquad \| R - R_0 \| \leqslant \delta_2; \qquad \| K_e - K_{e0} \| \leqslant \delta_3 \tag{15}$$

With the above in mind the adaptive robust control law forcing $\tilde{\mathbf{I}} = 0$ is then synthesized by

$$\mathbf{u} = L_0\dot{\mathbf{I}}_m + R_0\mathbf{I}_d + K_{e0}\dot{\mathbf{q}}_d - (\hat{\delta}_1 \| \dot{\mathbf{I}}_m \| + \hat{\delta}_2 \| \mathbf{I}_d \| + \hat{\delta}_3 \| \dot{\mathbf{q}}_d \| + \hat{\delta}_4 \| \hat{\alpha}_{ak} \| \, \| \dot{\mathbf{q}} \|)\,\mathrm{sgn}(\tilde{\mathbf{I}}) \tag{16}$$

$$\dot{\hat{\delta}}_1 = \eta_1 \| \dot{\mathbf{I}}_m \| \, \| \tilde{\mathbf{I}} \| \tag{17}$$

$$\dot{\hat{\delta}}_2 = \eta_2 \| \mathbf{I}_d \| \, \| \tilde{\mathbf{I}} \| \tag{18}$$

$$\dot{\hat{\delta}}_3 = \eta_3 \| \dot{\mathbf{q}}_d \| \, \| \tilde{\mathbf{I}} \| \tag{19}$$

$$\dot{\hat{\delta}}_4 = \eta_4 \| \hat{\alpha}_{ak} \| \, \| \dot{\mathbf{q}} \| \, \| \tilde{\mathbf{I}} \| \tag{20}$$

where $\mathbf{I}_d$ and $\dot{\mathbf{I}}_m$ are defined in (6) and (14) respectively, $\hat{\alpha}_{ak}$ is given by (8) and $\eta_i$ $(i = 1, 2, 3, 4)$ are constants determining the rate of the adaptations.

The structure of the controller given by (16) is sketched in Figure 1. The controller consists of two parts. In the first part, $I_d$ represents an *embedded* control input which may be viewed as an adaptive controller that ensures the convergence of tracking error if the actuator dynamics are not present. In the second part the input voltage $\mathbf{u}$ regulates the real armature currents about the *embedded* currents and therefore attempts to provide the control voltages necessary to make the desired motions.

The stability of the closed-loop system described by (1), (2), (6) and (16) is established in the following theorem.

 

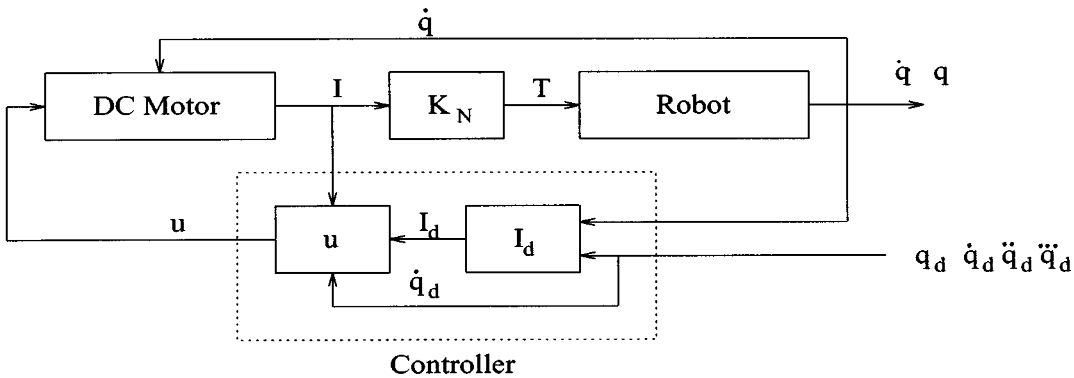


Figure 1. Control system

*Theorem 1*

If the robust control voltages $\mathbf{u}$ given by (6) and (16) are applied to the manipulator (1), (2), then all closed-loop signals are bounded and $\lim_{t\to\infty}\tilde{\mathbf{q}}=0$ provided that $\gamma$ initially satisfies

$$\gamma\lambda_q > 3\|B_d\| + \mu_1 + 2\vartheta\|\mathbf{q}_d\| + 2\vartheta\sqrt{\left(\frac{\lambda_{v2}}{\lambda_{v1}}\right)}\|\mathbf{x}_v(0)\| \tag{21}$$

where $\lambda_q$ is defined in (12), $\lambda_{v1}$, $\lambda_{v2}$ and $\mathbf{x}_v$ are defined in (34) and

$$\mu_1 \triangleq \frac{\beta_1^2}{4\lambda_r}, \qquad \beta_1 = (3+\varsigma+\|K_e\|), \qquad \lambda_r \triangleq \inf\frac{\tilde{\mathbf{I}}^{\mathrm{T}}R\tilde{\mathbf{I}}}{\|\tilde{\mathbf{I}}\|^2}$$

*Proof.* See Appendix II.

*Remarks*

1. The merit of the proposed algorithm, as is clear from the proof of Theorem 1, lies in the use of an adaptive compensator term $\hat{\delta}_4\|\hat{\alpha}_{ak}\|\,\|\dot{\mathbf{q}}\|\,\mathrm{sgn}(\tilde{\mathbf{I}})$ in the control law (16), which makes the use of $\dot{\mathbf{I}}_m$ instead of $\dot{\mathbf{I}}_d$ possible. In this case the stability of the closed loop can be guaranteed and the control algorithm is of the same complexity as the algorithm of Slotine and Li.[15]

2. It should be noted that the control law given by (16)–(20) depends on the choice of $\mathbf{I}_d$. In our design, $\mathbf{I}_d$ in (6) is composed of the feedback signal $\mathrm{I}_m$ and the feedforward signal $\mathbf{I}_f$. Therefore: (i) since the feedback signal $\mathrm{I}_m$ only involves the position feedback $\mathbf{q}$, the derivative of $\mathrm{I}_m$ only needs the velocity feedback $\dot{\mathbf{q}}$; (ii) the derivative of $\Phi_a(\mathbf{q}, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)$ in the feedforward signal $\mathbf{I}_f$ is only related to the velocity $\dot{\mathbf{q}}$, which is necessary for the compensator design. In this case the adaptive control law (16)–(20) for the cascade control system only requires the measurements of $\mathbf{I}$, $\mathbf{q}$ and $\dot{\mathbf{q}}$. This is the motivation for synthesizing $\mathbf{I}_d$ in (6). In addition, the removal of velocity feedback from the embedded controller can eliminate over parametrization in the actuator loop. This results in fewer update laws or bounding terms in the actuator loop.

3. There is a non-trivial difference between the adaptive law (16) and the control laws is References 4 and 6, which also only require the measurements of $\mathbf{I}$, $\mathbf{q}$ and $\dot{\mathbf{q}}$. Since the embeddd control laws for the manipulator loop, developed in References 4 and 6, involve the feedbacks of

 

$\mathbf{q}$ and $\dot{\mathbf{q}}$, derivative computation of the embedded control is not conducted. Instead, the knowledge of the upper bounds of the derivative of embedded control signals is used to avoid the requirement of acceleration feedback $\ddot{\mathbf{q}}$. In contrast, we compute $\|\dot{\mathbf{I}}_{\mathrm{m}}\|$ itself; therefore the upper bounds of $\dot{\mathbf{I}}_{\mathrm{d}}$ are not required in our scheme.

4. The control law (16) involves discontinuous functions and may result in chatter. However, in this case the chattering signal is the actuator voltage. As demonstrated in Reference 16, the torque signal is continuous after a lawpass filtering of the motor dynamics. From a practical point of view a chattering voltage is less difficult to synthesize and less prohibitive than a chattering torque, since many DC motors are controlled by pulse width modulation (PWM) signals. If the chattering effect is to be eliminated, this may be done by introducing a boundary layer at the expense of control accuracy. In our scheme it is easy to replace $\mathrm{sgn}(\tilde{\mathbf{I}})$ in (16) by

$$\pi(\tilde{\mathbf{I}}) = \begin{cases} \mathrm{sgn}(\tilde{\mathbf{I}}) & \text{if } \tilde{\mathbf{I}} > \varepsilon \\ \tilde{\mathbf{I}}/\varepsilon & \text{if } \tilde{\mathbf{I}} \leqslant \varepsilon \end{cases}$$

for some small $\varepsilon > 0$. However, the stability result changes. It is no longer asymptotically stable but can be shown to be uniformly ultimately bounded.

5. In this paper the bounds on $\delta_i$ $(i = 1, 2, 3, 4)$ are not assumed to be available and suitable integral updated laws are given so that the $\delta_i$ grow until they reach the levels necessary to compensate the non-linear dynamics.

## 3. A SIMULATION EXAMPLE

### *3.1. System Description*

As an illustration we will apply the adaptive algorithm (16)–(20) to a two-link robot arm with DC actuators proposed as a benchmark robotic system in Reference 14, shown in Figure 2. The robot model is described by (1) and (2). A parametrization scheme for this robot is given in Reference 6:

$$\begin{aligned} &\alpha_1 = m_2 l_1^2 + m_l l_1^2 + I_1 + I_2 + J_1 + I_l, \qquad \alpha_2 = I_2 + J_2 + I_l \\ &\alpha_3 = I_2 + I_l \qquad \alpha_4 = m_2 l_1 (l_{c2} + l_2) + m_l l_1 (l_{cl} + l_2) \\ &\alpha_5 = m_2 l_1 + m_1 (l_1 + l_{c1}) + m_l l_1, \qquad \alpha_6 = m_2 (l_2 + l_{c2}) + m_l (l_2 + l_{cl}) \end{aligned} \tag{22}$$

where $m_l$ is the mass of the end-effector and load, $I_l$ is the inertia of the end-effector and load, $l_{cl}$ is the mass $m_l$ center-of-gravity co-ordinate and $J_1$ and $J_2$ are the rotor inertias.

With this parametrization, $\Phi(\mathbf{q}, \dot{\mathbf{q}}_{\mathrm{d}}, \ddot{\mathbf{q}}_{\mathrm{d}})$ in (4) has components

$$\begin{aligned} &\phi_{11} = \ddot{q}_{d1}, \qquad \phi_{12} = 0 \\ &\phi_{13} = \ddot{q}_{d2}, \qquad \phi_{14} = \cos(q_2)(2\ddot{q}_{d1} + \ddot{q}_{d2}) - \sin(q_2)(\dot{q}_{d2}^2 + 2\dot{q}_{d1}\dot{q}_{d2}) \\ &\phi_{15} = g\cos(q_1), \qquad \phi_{16} = g\cos(q_1 + q_2) \\ &\phi_{21} = 0, \qquad \phi_{22} = \ddot{q}_{d2} \\ &\phi_{23} = \ddot{q}_{d1}, \qquad \phi_{24} = \cos(q_2)\ddot{q}_{d1} + \sin(q_2)\dot{q}_{d1}^2 \\ &\phi_{25} = 0, \qquad \phi_{26} = g\cos(q_1 + q_2) \end{aligned} \tag{23}$$

 

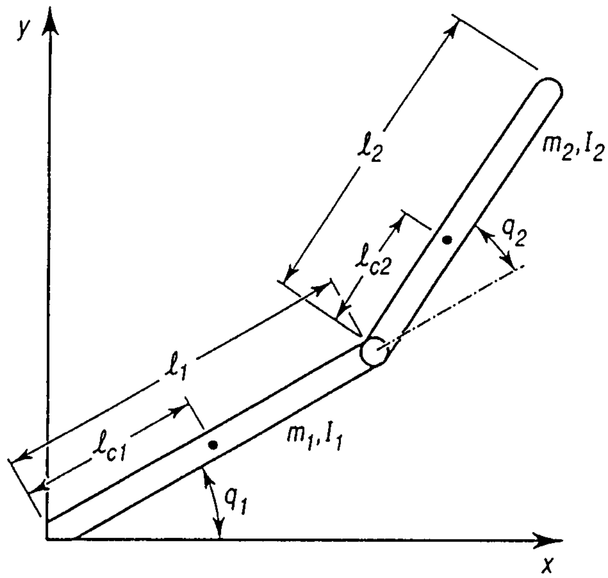


Figure 2. Two-linkage manipulator

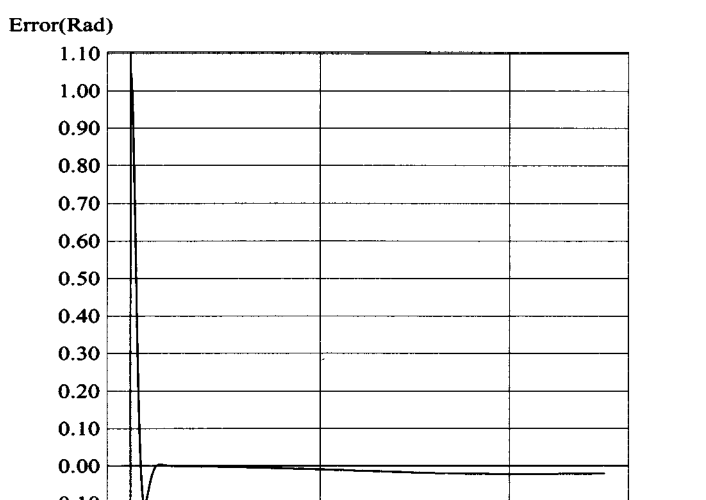


Figure 3. Tracking error of joint 1

The values of the manipulator and actuator parameters are given by[17] $l_1 = 0{\cdot}45$ m, $m_1 = 100$ kg, $l_{c1} = 0{\cdot}15$ m, $I_1 = 6{\cdot}25$ kg m$^2$, $J_1 = 4{\cdot}77$ kg m$^2$, $l_2 = 0{\cdot}20$ m, $m_2 = 25$ kg, $l_{c2} = 0{\cdot}10$ m, $I_2 = 0{\cdot}61$ kg m$^2$, $J_2 = 3{\cdot}58$ kg m$^2$, $m_l = 40$ kg, $l_{cl} = 0{\cdot}20$ m, $I_l = 7{\cdot}68$ kg m$^2$, $L_1 = 8 \times 10^{-5}$ V s A$^{-1}$,

 

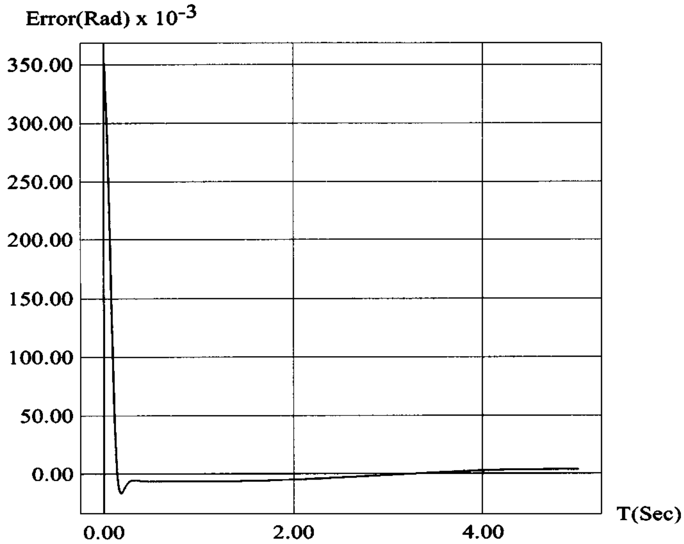


Figure 4. Tracking error of joint 2

$R_1 = 1{\cdot}5\ \Omega$, $K_{e1} = 25{\cdot}05$ V s, $K_{N1} = 25{\cdot}05$ V s, $L_2 = 8 \times 10^{-5}$ V s A$^{-1}$, $R_2 = 1{\cdot}5\ \Omega$, $K_{e2} = 21{\cdot}07$ V s and $K_{N2} = 21{\cdot}07$ V s.

We also need to choose the nominal system parameters. Let the uncertainty of the inertia parameters be originated by the varying load $m_l$. The electrical parameters are assumed to have 50% uncertainty. The nominal system parameters are given by $L_1 = 5 \times 10^{-5}$ V s A$^{-1}$, $R_1 = 1{\cdot}0\ \Omega$, $K_{e1} = 16.53$ V s, $L_2 = 5 \times 10^{-5}$ V s A$^{-1}$, $R_2 = 1{\cdot}0\ \Omega$, $K_{e2} = 14.54$ V s and $m_l = 20$ kg.

The desired $\mathbf{I}_d$ is synthesized by (6) with $\kappa = 8$, $\gamma^2 = 10$, $\Gamma = 15I$ and $\sigma = 0{\cdot}2$. The initial values of $\hat{\alpha}_{ak}$ are chosen as $\hat{\alpha}_{ak}(0) = [1{\cdot}0657, 0{\cdot}3575, 0{\cdot}1888, 0{\cdot}1051, 2{\cdot}1869, 2{\cdot}2911, 1{\cdot}2297, 0{\cdot}4125, 0{\cdot}2179, 0{\cdot}1213, 2{\cdot}5234, 2{\cdot}6434]^T$. The controller is then synthesized by (16) with $\eta_1 = 1 \times 10^{-11}$, $\eta_2 = 1 \times 10^{-6}$, $\eta_3 = 1 \times 10^{-6}$ and $\eta_4 = 1 \times 10^{-6}$. The initial values of $\hat{\delta}_i$ are chosen as $\hat{\delta}_1(0) = 8 \times 10^{-5}$, $\hat{\delta}_2(0) = 1$, $\hat{\delta}_3(0) = 10$ and $\hat{\delta}_4(0) = 10$.

### *3.2. Simulation results*

The control (16)–(20) is used to track the desired trajectories

$$q_{1d} = q_{2d} = -90^\circ + 52{\cdot}5[1 - \cos(1{\cdot}26t)]$$

The initial displacements and velocities are chosen as $q_1(0) = -30^\circ$, $q_2(0) = -70^\circ$ and $\dot{q}_1(0) = \dot{q}_2(0) = 0$. The results of the simulation are shown in Figure 3 and 4. Figure 3 shows the trajectory tracking error of joint 1. Figure 4 shows the trajectory tracking error of joint 2. The validity of this adaptive controller is confirmed for the purpose of trajectory tracking in the presence of actuator dynamics.

## 4. CONCLUSIONS

In this paper a backstepping-based hybrid adaptive control law has been derived incorporating both manipulator and actuator dynamics with uncertainties in both mechanical and electrical parameters. The control law requires the measurement only of joint positions, velocities and motor armature currents. Asymptotic stability of the closed-loop system is established in the Lyapunov sense. Simulations performed with a two-link example verified the validity of the algorithm.

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## APPENDIX I: Proof of Lemma 1

Consider a Lyapunov function candidate

$$L_a = \tfrac{1}{2}\,[\mathbf{x}^T \ \ \tilde{\alpha}_{ak}^T] \begin{bmatrix} P & 0 \\ 0 & (1/\sigma)E \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \tilde{\alpha}_{ak} \end{bmatrix} \tag{24}$$

where $E$ denotes the identity matrix. Given (24), one has

$$\lambda_1 \|\mathbf{x}_\alpha\|^2 \leqslant L_a \leqslant \lambda_2 \|\mathbf{x}_\alpha\|^2 \tag{25}$$

where $\mathbf{x}_\alpha^T \triangleq [\mathbf{x}^T \ \ \tilde{\alpha}_{ak}^T]$, $\lambda_1 \triangleq \frac{1}{2}\min\{\lambda_{\min}(P),\, 1/\sigma\}$ and $\lambda_2 \triangleq \frac{1}{2}\max\{\lambda_{\max}(P),\, 1/\sigma\}$.

The time derivative of $L_a$ is evaluated along the trajectory of (11) as

$$\dot{L}_a = -\mathbf{x}^T Q\mathbf{x} + \mathbf{x}^T PC(-K_N^{-1} B(\mathbf{q}, \dot{\mathbf{q}})\dot{\tilde{\mathbf{q}}} - K_N^{-1} B_d \dot{\tilde{\mathbf{q}}} + \Phi_a \tilde{\alpha}_{ak}) + \tfrac{1}{2}\mathbf{x}^T \dot{P}\mathbf{x} + \frac{1}{\sigma}\dot{\tilde{\alpha}}_{ak}^T \tilde{\alpha}_{ak} \tag{26}$$

When $\gamma \geqslant \max\{1, \kappa\}$, one can write

$$\begin{aligned} -\mathbf{x}^T PC K_N^{-1} B_d \dot{\tilde{\mathbf{q}}} &= -\left(\dot{\tilde{\mathbf{q}}} - \frac{1}{\gamma}\mathbf{w} + \frac{\kappa}{\gamma}\tilde{\mathbf{q}}\right)^T B_d \dot{\tilde{\mathbf{q}}} \\ &\leqslant 3\|B_d\| \; \|\mathbf{x}\|^2 \end{aligned} \tag{27}$$

$$\begin{aligned} \tfrac{1}{2}\mathbf{x}^T \dot{P}\mathbf{x} - \mathbf{x}^T PC K_N^{-1} B(\mathbf{q}, \dot{\mathbf{q}})\dot{\tilde{\mathbf{q}}} &= \frac{1}{\gamma}(\kappa\tilde{\mathbf{q}} - \mathbf{w})^T(\dot{D} - B(\mathbf{q}, \dot{\mathbf{q}})\dot{\tilde{\mathbf{q}}} \\ &\leqslant 2\vartheta\|\dot{\mathbf{q}}\| \; \|\mathbf{x}\|^2 \end{aligned} \tag{28}$$

where $\vartheta\|\dot{\mathbf{q}}\| = \dot{D} - B\|$ and the identity $\dot{\tilde{\mathbf{q}}}^T(\frac{1}{2}\dot{D} - B(\mathbf{q}, \dot{\mathbf{q}}))\dot{\tilde{\mathbf{q}}} = 0$ has been used to derive (28). Substituting (12), (27) and (28) into (26), one obtains

$$\begin{aligned} \dot{L}_a &\leqslant -(\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\dot{\mathbf{q}}\|)\|\mathbf{x}\|^2 + \left(\mathbf{z}^T\Phi_a + \frac{1}{\sigma}\dot{\tilde{\alpha}}_{ak}^T\right)\tilde{\alpha}_{ak} \\ &= -(\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\dot{\mathbf{q}}\|)\|\mathbf{x}\|^2 \end{aligned} \tag{29}$$

where the identity $\mathbf{x}^T PC\Phi_a\tilde{\alpha}_{ak} = \mathbf{z}^T\Phi_a\tilde{\alpha}_{ak}$ and equation (8) have been used.

 

From the definitions of $\mathbf{x}$ and $\mathbf{x}_\alpha$ it is easy to show from (25) that

$$\|\dot{\mathbf{q}}\| \leqslant \|\mathbf{q}_d\| + \|\mathbf{x}\| \leqslant \|\mathbf{q}_d\| + \|\mathbf{x}_\alpha\| \leqslant \|\mathbf{q}_d\| + \sqrt{\left(\frac{L_a}{\lambda_1}\right)} \tag{30}$$

which can be used to place an upper bound for $\dot{L}_a$ as follows:

$$\dot{L}_a \leqslant -\left[\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\mathbf{q}_d\| - 2\vartheta\sqrt{\left(\frac{L_a}{\lambda_1}\right)}\right]\|\mathbf{x}\|^2$$
$$\leqslant -\rho\|\mathbf{x}\|^2 \quad \text{for } L_a < \lambda_1\left(\frac{\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\mathbf{q}_d\|}{2\vartheta}\right)^2 \tag{31}$$

where $\rho$ is a positive constant. When $L_a < \lambda_1\ [(\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\mathbf{q}_d\|)/2\vartheta]^2$, $L_a$ is positive definite and $\dot{L}_a$ is negative semidefinite, we have $L_a(0) \geqslant L_a$ for all $t \geqslant 0$. From (25) we have $L_a(0) \leqslant \lambda_2\|\mathbf{x}_\alpha(0)\|^2$, which allows (31) to be written as

$$\dot{L}_a \leqslant -\rho\|\mathbf{x}\|^2 \quad \text{for } \lambda_2\|\mathbf{x}_\alpha(0)\|^2 < \lambda_1\left(\frac{\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\mathbf{q}_d\|}{2\vartheta}\right)^2 \tag{32}$$

which yields the gain condition of (13).

To complete the proof, it is necessary to show that $\tilde{\mathbf{q}} \to 0$ as $t \to \infty$. Since $\dot{L}_a$ is negative semidefinite, $\mathbf{x}$, and $\tilde{\alpha}_{ak}$ are all bounded, which implies that all signals on the right side of (11) are bounded. The boundedness of $\dot{\mathbf{x}}$ implies that $\mathbf{x}$ is uniformly continuous. Also, from (32) we can show that $\mathbf{x} \in \mathscr{L}_2^{3n}$. Therefore, as a direct consequence of Barbalat's lemma, we have $\lim_{t\to\infty}\mathbf{x} = 0$, which implies the result given in Lemma 1. □

## APPENDIX II: Proof of Theorem 1

The closed-loop stability is related to a Lyapunov function candidate

$$V(t) = L_a(t) + \bar{L}_i(t) \tag{33}$$

where $L_a(t)$ is defined in (24) and

$$\bar{L}_i(t) \triangleq \tfrac{1}{2}\tilde{\mathbf{I}}^T L\tilde{\mathbf{I}} + \tfrac{1}{2}\sum_{i=1}^{4}(\bar{\delta}_i - \hat{\delta}_i)^2/\eta_i$$

where $\bar{\delta}_1 = \delta_1$, $\bar{\delta}_2 = \delta_2$ and $\bar{\delta}_3 = \delta_3$, $\bar{\delta}_4 = \zeta$, $\delta_i$ $(i = 1, 2, 3)$ are defined in (15), $\zeta$ is defined in (40) and $\hat{\delta}_i$ are the estimates of $\bar{\delta}_i$. Given (33), one has

$$\lambda_{v1}\|\mathbf{x}_v\|^2 \leqslant V \leqslant \lambda_{v2}\|\mathbf{x}_v\|^2 \tag{34}$$

where $\mathbf{x}_v^T \triangleq [\mathbf{x}^T\ \tilde{\alpha}_{ak}^T\ \tilde{\mathbf{I}}^T\ (\bar{\delta}_1 - \hat{\delta}_1)\ \ldots\ (\bar{\delta}_4 - \hat{\delta}_4)]$, $\lambda_{v1} \triangleq \tfrac{1}{2}\min\{\lambda_{\min}(P), 1/\sigma, \lambda_{\min}(L), 1/\eta_i\ (i = 1 \ldots 4)\}$ and $\lambda_{v2} \triangleq \tfrac{1}{2}\max\{\lambda_{\max}(P), 1/\sigma, \lambda_{\max}(L), 1/\eta_i\ (i = 1 \ldots 4)\}$.

The time derivative of $L_a(t)$ should not be bounded from above by (29), since $\tilde{\mathbf{I}}$ is not necessarily an all-zero vector. Instead, an additional term $\mathbf{x}^T PC\tilde{\mathbf{I}}$ must be added to the right side of (29) to establish an upper bound for $\dot{L}_a$when $\tilde{\mathbf{I}} \neq 0$. As a result, one has to write

$$\dot{L}_a \leqslant -(\gamma\lambda_q - 3\|B_d\| - 2\vartheta\|\dot{\mathbf{q}}\|)\|\mathbf{x}\|^2 + \mathbf{x}^T PC\tilde{\mathbf{I}} \tag{35}$$

 

When $\gamma \geqslant \max\{1, \kappa\}$, one can write

$$\mathbf{x}^{\mathrm{T}} PC\tilde{\mathbf{I}} = \left(\dot{\tilde{\mathbf{q}}} - \frac{1}{\gamma}\mathbf{w} + \frac{\kappa}{\gamma}\tilde{\mathbf{q}}\right)^{\mathrm{T}} \tilde{\mathbf{I}} \leqslant 3\|\mathbf{x}\| \; \|\tilde{\mathbf{I}}\| \tag{36}$$

Consequently,

$$\dot{L}_{\mathrm{a}} \leqslant -(\gamma\lambda_q - 3\|B_{\mathrm{d}}\| - 2\vartheta\|\dot{\mathbf{q}}\|)\|\mathbf{x}\|^2 + 3\|\mathbf{x}\| \, \|\tilde{\mathbf{I}}\| \tag{37}$$

The time derivative of $L_{\mathrm{i}}(t)$ is evaluated along the trajectory (2) as

$$\dot{L}_{\mathrm{i}} = -\tilde{\mathbf{I}}^{\mathrm{T}}[L(\dot{\mathbf{I}}_{\mathrm{m}} + \dot{\mathbf{I}}_{\mathrm{f}}) + R\tilde{\mathbf{I}} + K_{\mathrm{e}}\dot{\tilde{\mathbf{q}}} - \mathbf{u} + R\mathbf{I}_{\mathrm{d}} + K_{\mathrm{e}}\dot{\mathbf{q}}_{\mathrm{d}}] + \sum_{i=1}^{3} (\bar{\delta}_i - \hat{\delta}_i)(-\dot{\hat{\delta}}_i)/\eta_i \tag{38}$$

When $\gamma \geqslant \max\{1, \kappa\}$, one can write

$$\begin{aligned} -\tilde{\mathbf{I}}^{\mathrm{T}} L\dot{\mathbf{I}}_{\mathrm{f}} &= -\tilde{\mathbf{I}}^{\mathrm{T}} L(\dot{\Phi}_{\mathrm{a}}\hat{\alpha}_{\mathrm{a}k} + \Phi_{\mathrm{a}}\dot{\hat{\alpha}}_{\mathrm{a}k}) \\ &\leqslant \alpha_l\|\tilde{\mathbf{I}}\|(\|\dot{\Phi}_{\mathrm{a}}\| \; \|\hat{\alpha}_{\mathrm{a}k}\| + \|\Phi_{\mathrm{a}}\| \; \|\dot{\hat{\alpha}}_{\mathrm{a}k}\|) \\ &\leqslant \alpha_l\|\tilde{\mathbf{I}}\|(\|\dot{\Phi}_{\mathrm{a}}\| \; \|\hat{\alpha}_{\mathrm{a}k}\| + 3\sigma\|\Phi_{\mathrm{a}}\|^2 \| \; \|\mathbf{x}\|) \end{aligned} \tag{39}$$

where $\alpha_l \triangleq \|L\|$ and equations (8) and (9) have been used. Since $\dot{\mathbf{q}}_{\mathrm{d}}$, $\ddot{\mathbf{q}}_{\mathrm{d}}$ and $\mathbf{q}_{\mathrm{d}}^{(3)}$ are uniformly bounded, one can write

$$\|\Phi_{\mathrm{a}}\| \leqslant \rho, \qquad \|\dot{\Phi}_{\mathrm{a}}\| \leqslant \varrho\|\dot{\mathbf{q}}\|$$

where $\rho$ and $\varrho$ are constants. Thus equation (39) becomes

$$\begin{aligned} -\tilde{\mathbf{I}}^{\mathrm{T}} L\dot{\mathbf{I}}_{\mathrm{f}} &\leqslant \alpha_l\varrho\|\tilde{\mathbf{I}}\| \; \|\dot{\mathbf{q}}\| \; \|\hat{\alpha}_{\mathrm{a}k}\| + 3\alpha_l\sigma\rho^2\|\tilde{\mathbf{I}}\| \; \|\mathbf{x}\| \\ &= \varsigma\|\tilde{\mathbf{I}}\| \; \|\mathbf{x}\| + \zeta\|\tilde{\mathbf{I}}\| \; \|\dot{\mathbf{q}}\| \; \|\hat{\alpha}_{\mathrm{a}k}\| \end{aligned} \tag{40}$$

where $\varsigma \triangleq 3\alpha_l\sigma\rho^2$ and $\zeta \triangleq \alpha_l\varrho$.

Substituting $\mathbf{u}$ in (38) by the control law (16) and noticing (15) and (40), one obtains

$$\begin{aligned} \dot{L}_i \leqslant & -\tilde{\mathbf{I}}^{\mathrm{T}} R\tilde{\mathbf{I}} - \tilde{\mathbf{I}}^{\mathrm{T}} K_{\mathrm{e}}\dot{\tilde{\mathbf{q}}} - \tilde{\mathbf{I}}^{\mathrm{T}} L\dot{\mathbf{I}}_{\mathrm{f}} + (\delta_1\|\dot{\mathbf{I}}_{\mathrm{m}}\| \; \|\tilde{\mathbf{I}}\| + \delta_2\|\mathbf{I}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\| + \delta_3\|\dot{\mathbf{q}}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\|) \\ & -(\hat{\delta}_1\|\dot{\mathbf{I}}_{\mathrm{m}}\| \; \|\tilde{\mathbf{I}}\| + \hat{\delta}_2\|\mathbf{I}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\| + \hat{\delta}_3\|\dot{\mathbf{q}}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\| + \hat{\delta}_4\|\hat{\alpha}_{\mathrm{a}k}\| \; \|\dot{\mathbf{q}}\| \; \|\tilde{\mathbf{I}}\|) + \sum_{i=1}^{4} (\bar{\delta}_i - \hat{\delta}_i)(-\dot{\hat{\delta}}_i)/\eta_i \\ \leqslant & \; \tilde{\mathbf{I}}^{\mathrm{T}} R\tilde{\mathbf{I}} - \tilde{\mathbf{I}}^{\mathrm{T}} K_{\mathrm{e}}\dot{\tilde{\mathbf{q}}} + (\delta_1\|\dot{\mathbf{I}}_{\mathrm{m}}\| \; \|\tilde{\mathbf{I}}\| + \delta_2\|\mathbf{I}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\| + \delta_3\|\dot{\mathbf{q}}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\|) + \varsigma\|\tilde{\mathbf{I}}\| \; \|\mathbf{x}\| + \zeta\|\tilde{\mathbf{I}}\| \; \|\dot{\mathbf{q}}\| \; \|\hat{\alpha}_{\mathrm{a}k}\| \\ & -(\hat{\delta}_1\|\dot{\mathbf{I}}_{\mathrm{m}}\| \; \|\tilde{\mathbf{I}}\| + \hat{\delta}_2\|\mathbf{I}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\| + \hat{\delta}_3\|\dot{\mathbf{q}}_{\mathrm{d}}\| \; \|\tilde{\mathbf{I}}\| + \hat{\delta}_4\|\hat{\alpha}_{\mathrm{a}k}\| \; \|\dot{\mathbf{q}}\| \; \|\tilde{\mathbf{I}}\|) + \sum_{i=1}^{4} (\bar{\delta}_i - \hat{\delta}_i)(-\dot{\hat{\delta}}_i)/\eta_i \\ \leqslant & -\tilde{\mathbf{I}}^{\mathrm{T}} R\tilde{\mathbf{I}} - \tilde{\mathbf{I}}^{\mathrm{T}} K_{\mathrm{e}}\dot{\tilde{\mathbf{q}}} + \varsigma\|\tilde{\mathbf{I}}\| \; \|\mathbf{x}\| \\ \leqslant & -\tilde{\mathbf{I}}^{\mathrm{T}} R\tilde{\mathbf{I}} + (\varsigma + \alpha_k)\|\mathbf{x}\| \; \|\tilde{\mathbf{I}}\| \end{aligned} \tag{41}$$

where $\alpha_k \triangleq \|K_{\mathrm{e}}\|$. Based on (37) and (41), $\dot{V}$ can be expressed as

$$\begin{aligned} \dot{V} &\leqslant -(\gamma\lambda_q - 3\|B_{\mathrm{d}}\| - 2\vartheta\|\dot{\mathbf{q}}\|)\|\mathbf{x}\|^2 + \beta_1\|\mathbf{x}\| \; \|\tilde{\mathbf{I}}\| - \tilde{\mathbf{I}}^{\mathrm{T}} R\tilde{\mathbf{I}} \\ &\leqslant -(\gamma\lambda_q - 3\|B_{\mathrm{d}}\| - 2\vartheta\|\dot{\mathbf{q}}\| - \mu_1)\|\mathbf{x}\|^2 - \lambda_r(\|\tilde{\mathbf{I}}\| - v_1\|\mathbf{x}\|)^2 \\ &\leqslant -(\gamma\lambda_q - 3\|B_{\mathrm{d}}\| - 2\vartheta\|\dot{\mathbf{q}}\| - \mu_1)\|\mathbf{x}\|^2 \end{aligned} \tag{42}$$

 

where

$$\beta_1 \triangleq (3 + \varsigma + \alpha_k), \qquad \mu_1 \triangleq \frac{\beta_1^2}{4\lambda_r}, \qquad v_1 \triangleq \frac{\beta_1}{2\lambda_r}, \qquad \lambda_r \triangleq \inf \frac{\tilde{\mathbf{I}}^{\mathrm{T}} R \tilde{\mathbf{I}}}{\| \tilde{\mathbf{I}} \|^2}$$

Similarly to the arguments in the proof of Lemma 1, $\dot{V}$ in (42) can be written as

$$\dot{V} \leqslant -\rho_v \| \mathbf{x} \|^2 \quad \text{for } \lambda_{v2} \| \mathbf{x}_v(0) \|^2 < \lambda_{v1} \left( \frac{\gamma\lambda_q - 3\| B_{\mathrm{d}} \| - 2\vartheta \| \mathbf{q}_{\mathrm{d}} \| - \mu_1}{2\vartheta} \right)^2 \tag{43}$$

where $\rho_v$ is a constant. Thus we obtain the gain condition of (21).

Following the same arguments as in the proof of Lemma 1, we can show that $\lim_{t\to\infty} \mathbf{x} = 0$, which implies the result given in Theorem 1. □

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