(6.4) is relatively easy and only lower dimensional matrix computations are involved.

2) Step 2 and Step 4 can be implemented simultaneously, which makes the algorithm attractive for parallel implementation.

VII. Conclusion

The notion of partial eigenstructure assignment (PEA), which is a natural extension of eigenstructure assignment and partial eigenvalue assignment, for linear multivariable systems has been introduced. Theoretical basis for PEA has been presented. Algorithms both for PEA and for eigenstructure assignment have been developed and analyzed. These algorithms have been shown to be especially effective for large scale system applications.

References

[1] W. L. Brogan, *Modern Control Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1985.

[2] H. Kimura, "Pole assignment by gain output feedback," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 509-516, Aug. 1975.

[3] B. C. Moore, "On the flexibility offered by state-feedback in multivariable systems beyond closed eigenvalue assignment," *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 689-692, Oct. 1976.

[4] G. Klein and B. C. Moore, "Eigenvalue-generalized eigenvector assignment with state feedback," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 140-141, Feb. 1977.

[5] V. Sinswat and F. Fallside, "Eigenvalue/eigenvector assignment by state-feedback," *Int. J. Contr.*, vol. 26, no. 3, pp. 369-403, 1977.

[6] B. Porter and J. J. D'Azzo, "Closed-loop eigenstructure assignment by state feedback in multivariable linear systems," *Int. J. Contr.*, vol. 27, no. 3, pp. 487-492, 1978.

[7] M. M. Fahmy and J. O'Reilly, "On eigenstructure assignment in linear multivariable systems," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 690-693, June 1982.

[8] —, "Eigenstructure assignment in linear multivariable systems—A parametric solution," *IEEE Trans. Automat. Contr.*, vol. AC-28, pp. 990-993, Oct. 1983.

[9] G. Roppeneck, "Minimum norm output feedback design under specific eigenvalue areas," *Syst. Contr. Lett.*, vol. 3, pp. 101-103, 1983.

[10] —, "On parametric state feedback design," *Int. J. Contr.*, vol. 43, no. 3, pp. 793-804, 1986.

[11] M. M. Hassan and M. H. Amin, "Recursive eigenvalue assignment in linear system," *Int. J. Contr.*, vol. 45, no. 1, pp. 291-310, 1987.

[12] M. A. Khan and N. Sreenivasulu, "A systematic method for eigenstructure assignment and application to response shaping," *Int. J. Contr.*, vol. 43, no. 2, pp. 717-735, 1986.

[13] J. W. Howze and R. K. Cavin, III, "Regular design with modal insensitivity," *IEEE Trans. Automat. Contr.*, vol. AC-24, pp. 466-469, Oct. 1979.

[14] M. M. Fahmy and J. O'Reilly, "Use of the design freedom of time-optimal control," *Syst. Contr. Lett.*, vol. 3, pp. 23-30, 1983.

[15] Y. Saad, "Projection and deflation methods for partial pole assignment in linear state feedback," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 290-296, Mar. 1988.

[16] M. J. Balas, "Trends in large structure control theory: Fondest dreams, widest hopes," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 522-535, Oct. 1982.

[17] G. H. Golub and D. F. Vanloan, *Matrix Computation*. Baltimore, MD: Johns Hopkins University Press, 1989.

[18] Ami Arbel, "Controllability measures and actuator placement in oscillatory systems," *Int. J. Contr.*, vol. 33, no. 3, pp. 565-574, 1980.

[19] R. Fletcher, *Practical Method of Optimization, Vol. 2*. New York: Wiley, 1987.

[20] J. E. Dennis, Jr., *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*. Englewood Cliffs, NJ: Prentice-Hall, 1983.

[21] J. J. More, B. S. Garbow, and K. E. Hillstorm, *User Guide for Minpack-1*.

An Adaptive Variable Structure Model Following Control Design for Robot Manipulators

Tin-Pui Leung, Qi-Jie Zhou, and Chun-Yi Su

Abstract—**An adaptive variable structure model following control design is presented for the nonlinear robot manipulator system. The controller does not require any knowledge of a nonlinear robotic system and does not necessarily need the occurrence of a sliding mode at each individually stable discontinuity surface. It thus greatly reduces the complexity of design. In the closed loop, the joint angles asymptotically converge to the reference trajectory with a prescribed transient response. The problem of chattering is reduced by the introduction of a boundary layer.**

I. Introduction

Adaptive model following control (AMFC) methodologies have recently received great attention in the robot manipulator control design (see, e.g., [1] for a recent review). Among developed AMFC algorithms, several approaches have been considered. Some use the Lyapunov stability method [2], [3], others do use the hyperstability theory [4], [5], and the deterministic approach [6], [7]. But the strict positive realness is invariably required. Furthermore, some of them can only guarantee the error between the states of the model and those of the controlled plant going to zero, the transient behavior of this error is not prescribed. A variable structure model following control (VSMFC) was proposed in [8], [9] as an alternative to adaptive model following control. The advantage of the VSMFC design lies in its ability to prescribe transient response requirements as well as providing a robust controller. But the control law, resulting from the method of control hierarchy [11] in order to force every trajectory to eventually come in contact with and remain on the intersection of n surfaces in joint space, is defined implicitly by a set of fairly complicated algebraic inequalities. Moreover, the control law is based on the restrictive assumption that the ranges of the variation of parameters are known and that the resulting control torques are excessive.

In this note, an adaptive variable structure model following control (AVSMFC) design is proposed for accomplishing trajectory tracking in a nonlinear robot system which ensures the stability of the intersection of the surfaces without necessarily stabilizing each individual one. Moreover, unlike [8], [9], we assume here that the system matrix is completely unknown and no information on its possible size is given. This has important implications. The involved computations of [8], [9] to obtain the feedback gains are not required here, instead, the feedback elements are generated as a function of state of the dynamics and the trajectory error. The approach avoids the difficulties linked to the strict positive realness requirement in traditional AMFC by taking advantage of the inherent positive definiteness of manipulators inertia matrix, and is easily extendable to a higher number of links. Chattering is eliminated by restricting the state of the system to slide within a boundary layer rather than along the intersection of the surfaces.

II. Robotic System

A. Manipulator Model

The dynamic equations of motion for a general rigid link manipu-

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T.-P. Leung is with the Department of Mechanical and Marine Engineering, Hong Kong Polytechnic, Hong Kong, People's Republic of China.

Q.-J. Zhou and C.-Y. Su are with the Department of Automation, South China University of Technology, Guangzhou 510641, People's Republic of China.

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lator having n degrees of freedom can be described as follows:

$$D(q)\ddot{q} + F(q,\dot{q})\dot{q} + G(q) = u(t) \quad (1)$$

where $q \in \mathbb{R}^n$ is joint displacement, $u \in \mathbb{R}^n$ is applied joint torque (or forces), $D(q) = D^T(q) > 0$, $D(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $F(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is the centripetal and Coriolis torques, and $G(q) \in \mathbb{R}^n$ is the gravitational torque.

Defining $x \in \mathbb{R}^{2n}$ to be the state vector

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}.$$

Equation (1) can be written in a state variable form

$$\dot{x} = \begin{bmatrix} \dot{q} \\ D^{-1}(q)(-F(q,\dot{q})\dot{q} - G(q)) \end{bmatrix} + \begin{bmatrix} 0 \\ D^{-1}(q) \end{bmatrix} u \quad (2a)$$

$$= \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} u \quad (2b)$$

$$= Ax + Bu. \quad (2c)$$

In AMFC design, the desired behavior of the plant is expressed through the use of a reference model driven by a reference model of the form

$$\frac{d}{dt}\begin{bmatrix} q_m \\ \dot{q}_m \end{bmatrix} = \begin{bmatrix} 0 & I \\ A_{m1} & A_{m2} \end{bmatrix}\begin{bmatrix} q_m \\ \dot{q}_m \end{bmatrix} + \begin{bmatrix} 0 \\ B_{m1} \end{bmatrix} r \quad (3a)$$

$$= A_m x_m + B_m r \quad (3b)$$

where $x_m = (q_m^T \;\; \dot{q}_m^T)^T \in \mathbb{R}^{2n}$, $A_m \in \mathbb{R}^{2n\times 2n}$, $B_m \in \mathbb{R}^{2n\times n}$ are constant matrices, $r \in \mathbb{R}^n$ is an external input.

B. AVSMFC Law

The considered tracking problem is stated as follows.

Knowing a reference input r, determine a control law u and a sliding surface such that sliding mode occurs on the sliding surface, the tracking error $e = x_m - x \in \mathbb{R}^{2n}$ has a prescribed transient response and it goes to zero asymptotically as $t \to \infty$.

We define the sliding surface $s \in \mathbb{R}^n$ as a hyperplane

$$s = Ge = 0 \quad (4)$$

where $G \in \mathbb{R}^{n\times 2n}$ is a constant matrix and will be determined as follows.

Let $e = (\epsilon^T \;\; \dot{\epsilon}^T)^T$, $\epsilon = q_m - q$, $\dot{\epsilon} = \dot{q}_m - \dot{q}$, then the sliding hyperplane becomes

$$s(\epsilon,\dot{\epsilon}) = Ge = (G_1 \;\; G_2)e = G_1\epsilon + G_2\dot{\epsilon} = 0. \quad (5)$$

Without losing generality, suppose $G_2 \in \mathbb{R}^{n\times n}$ is nonsingular. If the sliding mode exists on $s = 0$, then from the theory of variable structure system [10], the sliding mode is governed by the following linear differential equation whose behavior is dictated by the sliding hyperplane design matrices G_1 and G_2

$$\dot{\epsilon} = -G_2^{-1}G_1\epsilon. \quad (6)$$

Obviously, the tracking error transient response is then determined entirely by the eigenvector structure of the matrix $-G_2^{-1}G_1$. The approach of the determination of G_1, G_2 that ϵ has desired property is given in [11], [12]. Thus, if the control law is designed such that the sliding mode exists on $s = 0$, the tracking error transient response is completely governed by the linear dynamic equation (6).

In order to derive the AVSMFC law, we require the following assumption.

Assumption A1: For all $(x,t) \in \mathbb{R}^{2n} \times \mathbb{R}_+^1$, satisfy

$$(I - BB^+)B_m = 0 \quad (7a)$$

$$(I - BB^+)(A_m - A) = 0 \quad (7b)$$

$$(I - BB^+)(A_m + A_n) = 0 \quad (7c)$$

where

$$A_n = \begin{bmatrix} 0 & -I \\ G_2^{-1}G_1 & (I + G_2^{-1}G_1) \end{bmatrix} \quad (8)$$

and B^+ is pseudo inverse of B given by $B^+ = [0 \;\; B_1^{-1}]$.

Remark: We refer to (7a)–(7c) as matching conditions, which can always be satisfied due to the special structures of B, B_m, A, A_m, A_n, and B^+.

Define the AVSMFC law as the following:

$$u = K_1x + K_2r + K_3e + \psi_1x + \psi_2r + \psi_3e \quad (9)$$

where $k_1 \in \mathbb{R}^{n\times 2n}$, $K_2 \in \mathbb{R}^{n\times n}$, $K_3 \in \mathbb{R}^{n\times 2n}$ are constant matrices, $\psi_1 \in \mathbb{R}^{n\times 2n}$, $\psi_2 \in \mathbb{R}^{n\times n}$, $\psi_3 \in \mathbb{R}^{n\times 2n}$ are discontinuous matrices.

Differentiating s with respect to time gives sliding mode equation

$$\begin{aligned} \dot{s} &= G\dot{e} \\ &= G(A_mx_m + B_mr - Ax - Bu) \\ &= G(A_n(x - x_m) + (A_m + A_n)x_m \\ &\quad + B_mr - (A + A_n)x - Bu) \\ &= -s + GB\big[B^+(A_m - A) - K_1 - \psi_1)x \\ &\quad + (B^+B_m - K_2 - \psi_2)r \\ &\quad + (B^+(A_m + A_n) - K_3 - \psi_3)e\big]. \end{aligned} \quad (10)$$

Remark: The role of A_n is twofold. First, it satisfies $G = GA_n$, which makes it possible to use the Lyapunov function. Second, the matching condition (7c) can be satisfied.

We assume that the following condition is satisfied for all q, $\dot{q}$.

Assumption A2:

$$0 < \beta_1 \le \|G_2B_1\| \le \beta_2 < \infty \quad (11a)$$

$$\|B^+(A_m - A) - K_1\| < \sum_{i=1}^{3} \alpha_i\|e\|^{i-1} \quad (11b)$$

$$\|B^+B_m - K_2\| < \alpha_4 \quad (11c)$$

$$\|B^+(A_m + A_n) - K_3\| < \alpha_5 \quad (11d)$$

where $\alpha_i > 0$, $\beta_i > 0$ are some positive numbers.

Remark: Due to the mechanical characteristics of robotic manipulators and the boundness of the reference model, the assumption is valid.

We also make the following assumption.

Assumption A3: The minimum eigenvalue of matrix $G_2B_1G_2^T$ satisfies

$$\lambda_{\min}(G_2B_1G_2^T) \ge \beta_1. \quad (12)$$

Remark: Assumption A3 can always be held for suitable β_1.

Now consider the control law (9), if we take ψ_i as

$$\psi_1 = \begin{cases} \sum_{i=1}^{3} \hat{c}_i\|e\|^{i-1}\dfrac{G_2^Ts}{\|s\|}\operatorname{sgn}(x)^T & \|s\| \ne 0 \\ 0 & \|s\| = 0 \end{cases} \quad (13a)$$

$$\psi_2 = \begin{cases} \hat{c}_4\dfrac{G_2^Ts}{\|s\|}\operatorname{sgn}(r)^T & \|s\| \ne 0 \\ 0 & \|s\| = 0 \end{cases} \quad (13b)$$

$$\psi_3 = \begin{bmatrix} \hat{c}_5 \dfrac{G_3^T s}{\|s\|} \operatorname{sgn}(e)^T & \|s\| \neq 0 \\ 0 & \|s\| = 0 \end{bmatrix} \tag{13c}$$

$$\dot{\hat{c}}_j = g_j \|e\|^{j-1} \|s\| \sum_{i=1}^{2n} |x_i|, \qquad i = 1,2,3 \tag{14a}$$

$$\dot{\hat{c}}_4 = g_4 \|s\| \sum_{i=1}^{n} |r_i| \tag{14b}$$

$$\dot{\hat{c}}_5 = g_5 \|s\| \sum_{i=1}^{2n} |e_i| \tag{14c}$$

where $g_i > 0$, $i = 1 \cdots 5$ are arbitrary constant numbers.

We have the following theorem.

Theorem: Consider robotic system (1) with sliding surface (4) and control laws (9), (13), (14), satisfying Assumptions A1, A2, and A3, then the tracking error e converges to the sliding surface and is restricted to the surface for all subsequent times.

Proof: Consider the following Lyapunov function:

$$V(s) = \frac{1}{2} s^T s + \frac{1}{2} \left\{ \sum_{i=1}^{5} (c_i - \hat{c}_i)^2 / g_i \right\} \beta_1 \tag{15}$$

where c_i are numbers satisfying $c_i \geq \alpha_i \beta_2 / \beta_1$ and $\hat{c}_i$ is its estimation.

Differentiating $V(s)$ with respect to time t and using (10)–(14), we can observe that

$$(s^T G_2 B_1 G_2^T s) \geq \lambda_{\min}(G_2 B_1 G_2^T) \|s\|^2$$

then yields

$$\begin{aligned}
\dot{V}(t) &= s^T \dot{s} + \left\{ \sum_{i=1}^{5} (c_i - \hat{c}_i)(-\dot{\hat{c}}_i)/g_i \right\} \beta_1 \\
&= -s^T s + s^T (G_2 B_1) \big[(B^+ (A_m - A) - K_1) x \\
&\quad + (B^+ B_m - K_2) r + (B^+ (A_n + A_m) - K_3) e \big] \\
&\quad - s^T (G_2 B_1) \left\{ \sum_{j=1}^{3} \hat{c}_j \|e\|^{i-1} \frac{G_2^T s}{\|s\|} \operatorname{sgn}(x)^T x \right. \\
&\quad \left. + \hat{c}_4 \frac{G_2^T s}{\|s\|} \operatorname{sgn}(r)^T r + \hat{c}_5 \frac{G_2^T s}{\|s\|} \operatorname{sgn}(e)^T e \right\} \\
&\quad - \sum_{j=1}^{3} (c_j - \hat{c}_j) \|e\|^{j-1} \|s\| \sum_{i=1}^{2n} |x_i| \beta_1 \\
&\quad - (c_4 - \hat{c}_4) \|s\| \sum_{i=1}^{n} |r_i| \beta_1 - (c_5 - \hat{c}_5) \|s\| \sum_{i=1}^{2n} |e_i| \beta_1 \\
&\leq -s^T s + \|s\| \beta_2 \sum_{j=1}^{3} \alpha_j \|e\|^{j-1} \sum_{i=1}^{2n} |x_i| \\
&\quad + \|s\| \beta_2 \alpha_4 \sum_{i=1}^{n} |r_i| + \|s\| \beta_5 \alpha_5 \sum_{i=1}^{2n} |e_i| \\
&\quad - \|s\| \beta_1 \sum_{j=1}^{3} c_j \|e\|^{j-1} \sum_{i=1}^{2n} |x_i| - \|s\| \beta_1 c_4 \sum_{i=1}^{n} |r_i| \\
&\quad - \|s\| \beta_1 c_5 \sum_{i=1}^{2n} |e_i| + \sum_{j=1}^{3} \hat{c}_j \|e\|^{j-1} \|s\| \sum_{i=1}^{2n} |x_i| \beta_1 \\
&\quad + \hat{c}_4 \|s\| \sum_{i=1}^{n} |r_i| \beta_1 + \hat{c}_5 \|s\| \sum_{i=1}^{2n} |e_i| \beta_1 \\
&\quad - \lambda_{\min}(G_2 B_1 G_2^T) \\
&\quad \cdot \left\{ \sum_{j=1}^{3} \hat{c}_j \|e\|^{j-1} \|s\| \sum_{i=1}^{2n} |x_i| \right. \\
&\quad \left. + \hat{c}_4 \|s\| \sum_{i=1}^{n} |r_i| + \hat{c}_5 \|s\| \sum_{i=1}^{2n} |e_i| \right\} \\
&\leq -s^T s < 0.
\end{aligned}$$

Hence, the theorem is proved.

This, in turn, implies that $q \to q_m$, $\dot{q} \to \dot{q}_m$ as $t \to \infty$. Hence, the AVSMFC defined by (9), (13), and (14) is globally asymptotically stable and guarantees zero tracking error.

Remark:

1) In the theorem, we avoid the difficulties linked to the strict positive realness requirement in traditional AMFC by taking advantage of the inherent positive definiteness of the manipulators' inertia matrix.

2) The matrices K_i can be chosen in such a way so as to reduce the lower bounds on α_i. This, in turn, reduces the magnitudes of c_i. Consequently, the energy required by the adaption mechanism in generation ψ_i is minimized.

C. Insertion of the Boundary Layer

While assuring the desired behavior, the control law (9) is discontinuous across the sliding surface s, which leads to control chattering. Chattering is, in general, highly undesirable in practice, since it involves extremely high control activity and, further, may excite high-frequency dynamics neglected during modeling [13]. We can remedy this situation by smoothing out the control discontinuities in a boundary layer neighboring the sliding surface.

Consider the vector $\delta = (\delta_1 \quad \delta_2 \quad \delta_3)$ where $\delta_i > 0$ for $i = 1, 2, 3$. Let us replace ψ_i in (13) by $\psi_2(\delta)$ where

$$\psi_1 = \begin{bmatrix} \displaystyle\sum_{i=1}^{3} \hat{c}_i \|e\|^{i-1} \frac{G_2^T s}{\|s\|} \operatorname{sgn}(x)^T & \text{if } \|s\| > \delta_1 \\ \displaystyle\sum_{i=1}^{3} \hat{c}_i \|e\|^{i-1} \frac{G_2^T s}{\delta_1} \operatorname{sgn}(x)^T & \text{if } \|s\| \leq \delta_1 \end{bmatrix} \tag{16a}$$

$$\psi_2 = \begin{bmatrix} \hat{c}_4 \dfrac{G_2^T s}{\|s\|} \operatorname{sgn}(r)^T & \text{if } \|s\| > \delta_2 \\ \hat{c}_4 \dfrac{G_2^T s}{\delta_2} \operatorname{sgn}(r)^T & \text{if } \|s\| \leq \delta_2 \end{bmatrix} \tag{16b}$$

$$\psi_3 = \begin{bmatrix} \hat{c}_5 \dfrac{G_2^T s}{\|s\|} \operatorname{sgn}(e)^T & \text{if } \|s\| > \delta_3 \\ \hat{c}_5 \dfrac{G_2^T s}{\delta_3} \operatorname{sgn}(e)^T & \text{if } \|s\| \leq \delta_3. \end{bmatrix} \tag{16c}$$

By applying the AVSMFC laws (9), (16), (14), we will guarantee the attractiveness of the boundary layer. For the region inside the boundaries, it can be proved that this will guarantee the ultimate boundedness of the system to within any neighborhood of the boundary layer [14].

III. Simulation Example

Fig. 1 shows a 2-link robotic manipulator model used by [15]. The dynamic equation is given by

$$\begin{bmatrix} D_{11}(\phi) & D_{12}(\phi) \\ D_{12}(\phi) & D_{22}(\phi) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} F_{12}(\phi)\dot{\phi}^2 + 2F_{12}(\phi)\dot{\theta}\dot{\phi} \\ -F_{12}(\phi)\dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} q_1(\theta, \phi) g \\ q_2(\theta, \phi) g \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{17}$$

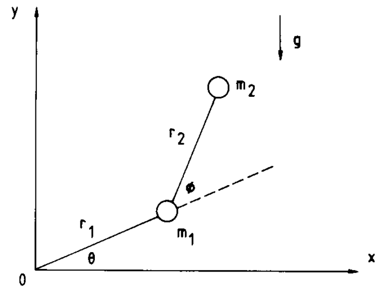

Fig. 1. Two-link robotic manipulator model.

where

$$D_{11}(\phi) = (m_1 + m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos(\phi) + J_1$$
$$D_{12}(\phi) = m_2 r_2^2 + m_2 r_1 r_2 \cos(\phi)$$
$$D_{22}(\phi) = m_2 r_2^2 + J_2$$
$$F_{12}(\phi) = m_2 r_1 r_2 \sin(\phi)$$
$$q_1(\theta, \phi) = -((m_1 + m_2) r_1 \cos(\phi) + m_2 r_2 \cos(\phi + \theta))$$
$$q_2(\theta, \phi) = -m_2 r_2 \cos(\theta + \phi). \tag{18}$$

The parameter values used are the same as those in [15].

$$r_1 = 1\text{ m}, \quad r_2 = 0.8\text{ m}$$
$$J_1 = 5\text{ kg}\cdot\text{m}, \quad J_2 = 5\text{ kg}\cdot\text{m}$$
$$m_1 = 0.5\text{ kg}, \quad m_2 = 6.25\text{ kg}.$$

A reference model is chosen such that the desired behavior of the arm motion is expressed by two decoupled linear systems

$$\theta_{ri} = A_{ri}\theta_{ri} + B_{ri}r_i, \qquad i = 1, 2 \tag{19}$$

where

$$A_{ri} = \begin{bmatrix} 0 & 1 \\ a_{1i} & a_{2i} \end{bmatrix}, \qquad B_{ri} = \begin{bmatrix} 0 \\ b_i \end{bmatrix}. \tag{20}$$

We select the following set of reference model parameters:

$$a_{11} = a_{12} = a_{21} = a_{22} = -1$$
$$b_1 = b_2 = 1.$$

The rise time and the settling time of the reference model are 1.9 s and 8 s, respectively. We choose driving inputs to the reference model to be constants.

The goal of the AVSMFC design is to force the tracking errors $\epsilon_1 = \theta_{r1} - \theta$ and $\epsilon_2 = \theta_{r2} - \phi$ to slide along the sliding surfaces. We choose these sliding surfaces:

$$\begin{aligned} s_1 &= c_{11}\epsilon_1 + \dot{\epsilon}_1 \\ s_2 &= c_{21}\epsilon_2 + \dot{\epsilon}_2 \end{aligned} \tag{21}$$

where $c_{11} = c_{21} = 5$.

The resulting sliding mode equations are two decoupled first-order systems

$$\dot{\epsilon} = -c_i\epsilon_i, \qquad i = 1, 2. \tag{22}$$

Since we are interested in trajectory tracking, we consider a situation characterized by the same initial condition on the reference model state and on the plant state. For this example, we pick the initial displacements and velocities to be

$$\theta_{r1}(t_0) = \theta(t_0) = -1.57, \theta_{r2}(t_0) = \phi(t_0) = 0$$
$$\dot{\theta}_{r1}(t_0) = \dot{\theta}_{r2}(t_0) = \dot{\theta}(t_0) = \dot{\phi}(t_0) = 0. \tag{23}$$

The K_i $(i = 1, 2, 3)$ in (9) are chosen so as to reduce the lower bounds of α_i in (11). Here, due to the complexity of the matrix A,

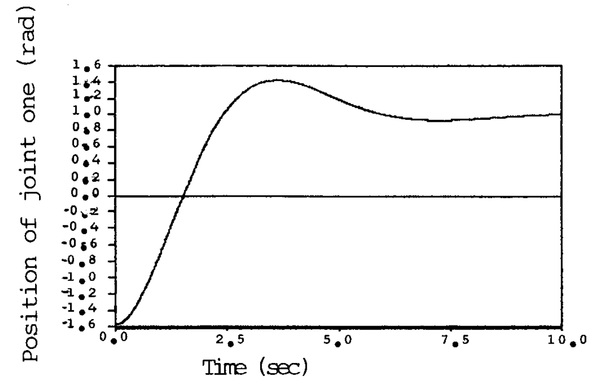

Fig. 2. Time response of joint one.

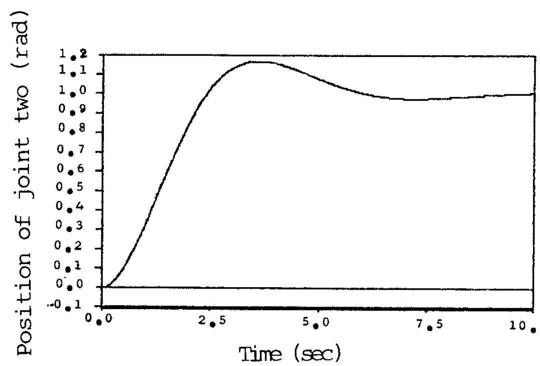

Fig. 3. Time response of joint two.

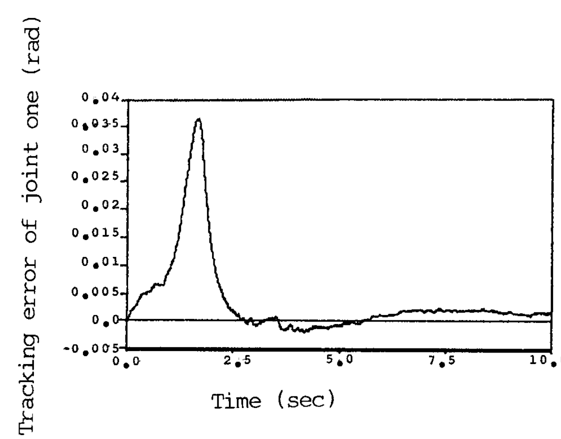

Fig. 4. Tracking error of joint one.

we simply choose

$$K_1 = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 15.75 & 4 \\ 4 & 9 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} 67.5 & 17 & 75 & 20 \\ 17 & 40 & 20 & 45 \end{bmatrix} \tag{24}$$

we took $\hat{c}_i(t_0)$ $(i = 1 \cdots 5)$ in control law (14) and reference inputs as

$$\hat{c}_1(t_0) = \hat{c}_2(t_0) = \hat{c}_3(t_0) = 25, \hat{c}_4(t_0) = 10, \hat{c}_5(t_0) = 30 \tag{25}$$
$$r_1 = r_2 = 1. \tag{26}$$

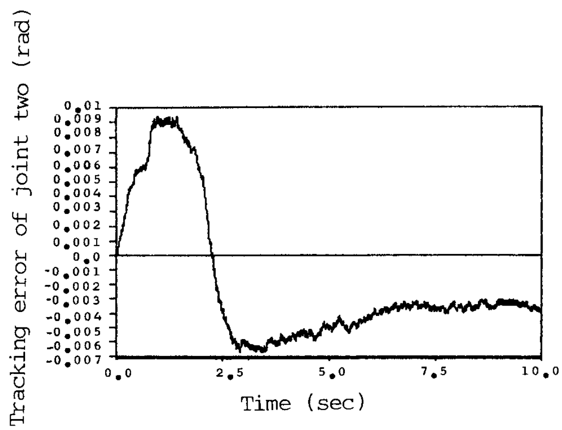

Fig. 5. Tracking error of joint two.

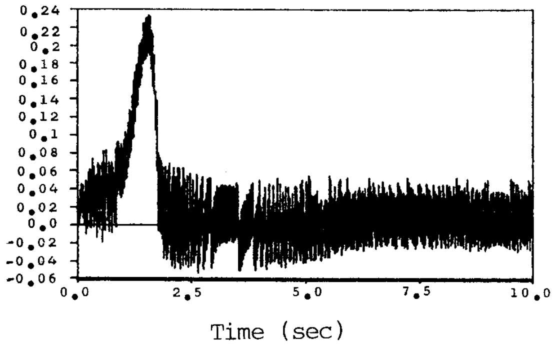

Fig. 6. Sliding surface s_1.

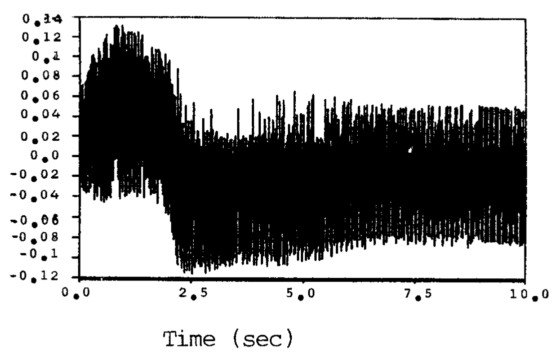

Fig. 7. Sliding surface s_2.

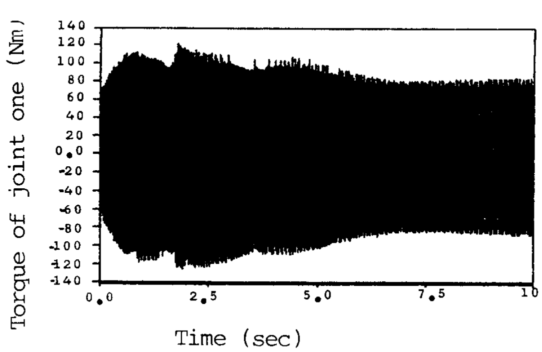

Fig. 8. Torque developed at joint one.

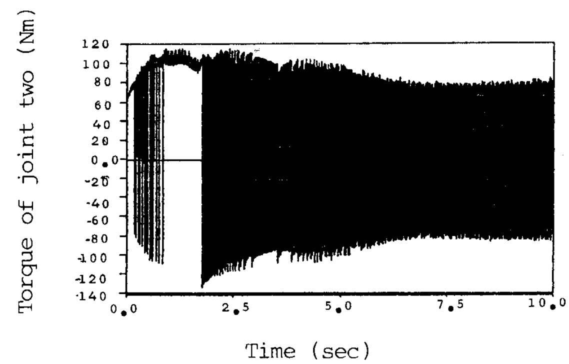

Fig. 9. Torque developed at joint two.

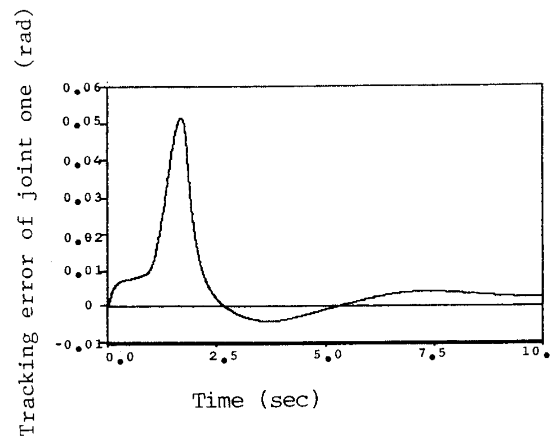

Fig. 10. Tracking error of joint one with boundary layer.

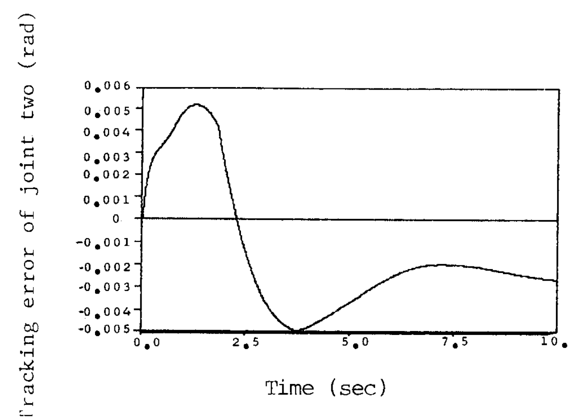

Fig. 11. Tracking error of joint two with boundary layer.

Using control laws (9), (13), (14), Figs. 2 and 3 show the time responses. Figs. 4 and 5 show the tracking errors. Figs. 6 and 7 show the sliding surface which imply the AVSMFC law achieves its objective successfully. Figs. 8 and 9 show torques developed at manipulator joints which result in undesirable robot chattering.

To reduce the chattering, we will implement the boundary layer AVSMFC scheme given in (9), (16), and (14). Here, we took $\delta_1 = \delta_2 = 0.05$. Figs. 10 and 11 show the tracking errors. Figs. 12 and 13 show sliding sectors. Figs. 14 and 15 show the torques exerted at manipulator joints. As can be seen from these figures, chattering is almost eliminated.

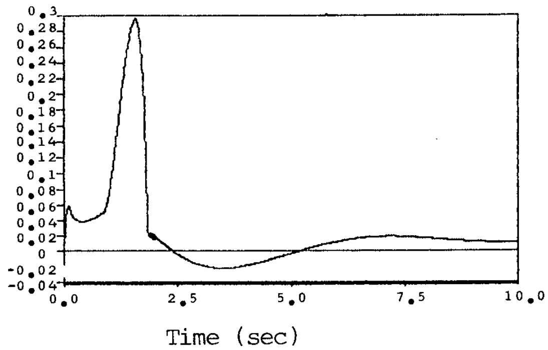

Fig. 12. Sliding surface s_1 with boundary layer.

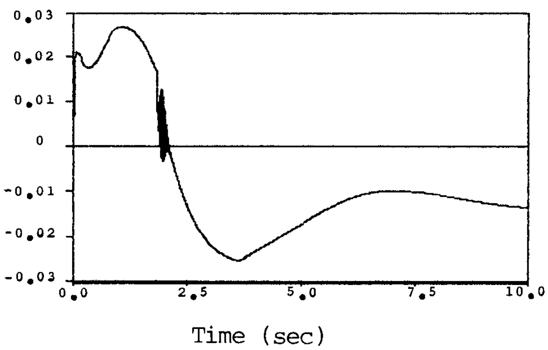

Fig. 13. Sliding surface s_2 with boundary layer.

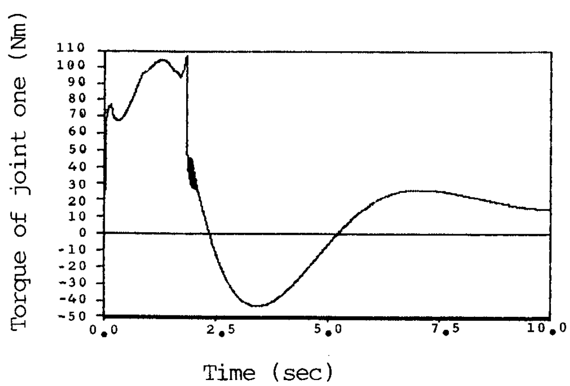

Fig. 14. Torque developed at joint one with boundary layer.

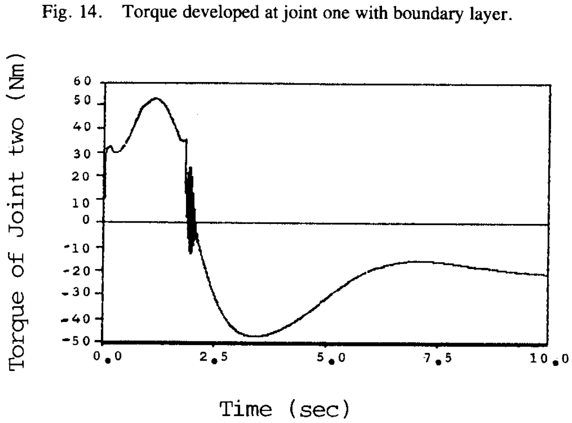

Fig. 15. Torque developed at joint two with boundary layer.

IV. Conclusion

An adaptive variable structure model following control design methodology is presented by using the theory of VSS. The major contribution of this methodology lies in the introduction of a special matrix, which makes it possible for using the Lyapunov function such as $V = s^T s$ instead of the requirement $s_i \dot{s}_i < 0$, and the derivation of the VSS controller does not require any knowledge of nonlinear robotic systems by adaptation of scalar gain. Chattering during the transient phase can be reduced by using the boundary layer technique. Simulation results show the good performance of the robot system.

References

[1] T. C. Hsia, "Adaptive control of robot manipulators—A review," in *Proc. IEEE Int. Conf. Robot. Automat.*, 1986, p. 183.

[2] S. Dubowsky and D. T. DesForses, "The application of model referenced adaptive control to robot manipulators," *Trans. ASME J. Dynam. Syst., Measur., Contr.*, vol. 101, pp. 193–200, Sept. 1979.

[3] H. Durrant-Shyte, "Practical adaptive control of actuated spatial mechanisms," in *Proc. IEEE Int. Conf. Robot. Automat.*, 1985, p. 650.

[4] A. Balestrino, G. DeMaria, and L. Sciavicco, "An adaptive model following control for robot manipulators," *Trans. ASME J. Dynam. Syst., Measur., Contr.*, vol. 105, pp. 143–152, Sept. 1983.

[5] S. Nicosia and P. Tomei, "Model reference adaptive control algorithms for industrial robots," *Automatica*, vol. 20, pp. 635–644, 1984.

[6] S. N. Singh, "Adaptive model following control of nonlinear robotic systems," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 1099–1100, 1985.

[7] Y. H. Chen, "Adaptive robust model-following control and application to robot manipulators," *Trans. ASME J. Dynam. Syst., Measur., Contr.*, vol. 109, pp. 209–215, Sept. 1987.

[8] K. D. Young, "A variable structure model following control design for robotic application," *IEEE J. Robot. Automat.*, vol. 4, pp. 556-561, 1988.

[9] L. Guzzela and H. P. Geering, "Model following variable structure control for a class of uncertain mechanical systems," in *Proc. 25th Conf. Decision Contr.*, 1986, p. 312.

[10] V. I. Utkin, *Sliding Modes and Their Applications.* Moscow: Mir, 1978.

[11] O. M. E. El-Ghezawi, A. S. Zinober, and S. A. Billings, "Analysis and design of variable structure system using a geometric approach," *Int. J. Contr.*, vol. 38, pp. 657-671, 1983.

[12] C. M. Dorling and A. S. I. Zinober, "Robust hyperplane design in multivariable variable structure control systems," *Int. J. Contr.*, vol. 48, pp. 2043–2054, 1988.

[13] J.-J. E. Slotine and S. S. Sastry, "Tracking control of nonlinear system using sliding surface, with application to robot manipulators," *Int. J. Contr.*, vol. 38, pp. 465-492, 1983.

[14] M. J. Corless and G. Leitmann, "Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic system," *IEEE Trans. Automat. Contr.*, vol. AC-26, pp. 1139–1144, 1981.