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A Sliding Mode Controller with Improved Adaptation Laws for the Upper Bounds on the Norm of Uncertainties*

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Key Words—Variable structure control; sliding mode control; adaptation; stability; robustness; uncertainty.

Abstract—In this paper an improved adaptation law on the upper bound of uncertainties is proposed to guarantee the boundedneness of both states of the plant and the estimated control gains when the boundary layer technique is employed. © 1998 Elsevier Science Ltd. All rights reserved.

1. *Introduction*

Recently, sliding mode control (SMC) has been developed as a useful strategy to be implemented with uncertain systems. When the state is constrained to the sliding surface, sliding mode control can completely reject uncertainties which satisfy the matching condition (Utkin, 1978). In sliding mode control an important assumption is that the uncertainties are bounded and that their bounds are available to the designer. These bounds are an important clue to the possibility of guaranteed stability of a closed-loop system. Occasionally, due to the complexity of the structure of uncertainties, their bounds may not be easily obtained. Adaptive control is also known as an effective and robust strategy with uncertain systems. Using online identification, one can assure global stability for a class of systems whose structure is known but whose parameters are unknown, however researchers have noted poor performance with unmodeled uncertainties. Studies have been done, and are currently being done to discover ways to recover stability in these systems with unmodeled uncertainties. The challenge addressed in this paper is to combine the strengths of both these approaches and provide adaptive sliding mode control.

Recently, many kinds of controllers which combine the structure of sliding mode control and adaptive control have been presented. Therein Leung *et al.* (1991), Yoo and Chung (1992), Su and Leung (1993), Chen and Mita (1993) and Parra-Vega *et al.* (1994) proposed sliding mode controllers in which the control gains switch with the estimated adapting upper bound of the matching uncertainties. As is well known, a draw back of sliding mode controllers is the discontinuity about the switching surface. For practical implementations the controller must be smoothed. One way to smooth the control law is to introduce a boundary layer about the switching surface as in Slotine and Sastry (1983). In this case, however, we believe the results of Chen and Mita (1993), Leung *et al.* (1991) and Yoo and Chung (1992) may be questionable because if the boundary layer is used with the proposed control law, the estimated switching gain can grow unboundedly in the boundary layer since the restriction to the sliding surface cannot always be achieved.

For completion of this method, in this paper, we propose an improved adaptive law for the upper bound of the norm of the uncertainties. Stability analysis shows that this method guarantees the boundedness of both the state of the plant and the adaptive gain, when boundary layer techniques are employed. Simulation results are shown and the results verify our method.

2. *System model description*

Consider a dynamic system described by

$$\dot{x}(t) = (A + \delta A(t,x))x(t) + (B + \delta B(t,x))u(t) + C(t,x)v(t) + f(t,x), \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control, $v(t) \in \mathbb{R}^l$ is an extraneous disturbance, and A, B, and $C(t,x)$ are matrices of appropriate dimensions with B of full rank. $\delta A(t,x)$, $f(t,x)$, and $\delta B(t,x)$ represent the uncertainty of the linear portion, the nonlinearity of the system, and the input matrix uncertainty, respectively.

To complete the description of the uncertain dynamical system, the following standard assumptions are introduced.

Assumption 2.1:

(i) For existence purposes, $\delta A(\cdot,\cdot)$, $f(\cdot,\cdot)$, $\delta B(\cdot,\cdot)$, $C(\cdot,\cdot)$, and $v(\cdot)$ are continuous on their arguments.

(ii) *Matching conditions*: There exists functions $D(\cdot,\cdot)$: $\mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^{m\times n}$, $E(\cdot,\cdot)$: $\mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^m$, $F(\cdot,\cdot)$: $\mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^{m\times m}$, and $G(\cdot,\cdot)$: $\mathbb{R}\times\mathbb{R}^n \to {}^{m\times l}$, such that

$$\delta A(t,x) = BD(t,x),$$
$$f(t,x) = BE(t,x),$$
$$\delta B(t,x) = BF(t,x),$$
$$C(t,x) = BG(t,x) \text{ for all } (t,x) \in \mathbb{R}\times\mathbb{R}^n. \qquad (2)$$

(iii) The pair (A, B) is completely controllable.

Assume that a sliding mode control is employed for controlling the system under consideration. From the structural assumption, all uncertain elements can be *lumped* and the system (1) can be written as

$$\dot{x}(t) = Ax(t) + B(u(t) + e(t,x)), \qquad (3)$$

where $e(t,x)$ is the *lumped* uncertainty. Based solely on the knowledge of the bound on the uncertainty, we introduce the following assumption.

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Assumption 2.2. There exists a continuous positive scalar valued function $\rho(\cdot,\cdot): \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}_+$, such that $\|e(t,x)\| \le \rho(t,x)$ for all $(t,x) \in \mathbb{R} \times \mathbb{R}^n$.

SMC design is broken down into two phases (Utkin, 1978). The first phase entails constructing a switching surface so that the system restricted to the switching surface produces a desired behavior (Utkin and Young, 1978). We define the $n-m$ dimensional subspace Ω (the switching space) as follows:

$$\Omega \triangleq \{x : \sigma_i(x) \triangleq S_i x = 0,\ S_i \in \mathbb{R}^{m\times n},\ i = (1,\ldots,m),\ x \in \mathbb{R}^n\}. \tag{4}$$

Let $S = [s_1^T, \ldots, s_m^T]^T \in \mathbb{R}^{m\times n}$, and $\sigma(x) = [\sigma_1, \ldots, \sigma_m]^T \in \mathbb{R}^m$. Without loosing generality we assume that the matrix S is of full rank and the matrix SB is nonsingular.

After selecting the switching surface, the next step is to choose the control law such that the condition $\sigma^T\dot{\sigma} < 0$ is satisfied. This condition assures us that the switching surface will attract the system trajectories and that upon the intersection with the switching surface the trajectories will remain there for all following time. For convenience, throughout this paper, the arguments t and x are sometimes omitted when no confusion is likely to arise.

3. *Design of a sliding mode controller*

We consider the following SMC law:

$$u = -(SB)^{-1}K\sigma + u_{eq_{nom}} + u_N, \tag{5}$$

where $K \in \mathbb{R}^{m\times m}$ is a positive definite matrix, $u_{eq_{nom}}$ is the equivalent control for the nominal system of equation (3) by assuming that the uncertainty $e(t,x)$ is zero, which is obtained by solving the following equation:

$$\dot{\sigma}_{nom} = SAx + SBu_{eq_{nom}} = 0. \tag{6}$$

The equivalent control is determined by

$$u_{eq_{nom}} = -(SB)^{-1}SAx. \tag{7}$$

This equivalent control determines the behavior of the nominal system restricted to the switching surface. The term u_N represents the nonlinear feedback control for suppression of the effect of the uncertainty and drives the system trajectories toward the switching surface until intersection occurs

$$u_N = \begin{cases} -\dfrac{B^TS^T\sigma}{\|B^TS^T\sigma\|}\rho(t,x), & \text{if } \sigma \neq 0, \\ 0 & \text{if } \sigma = 0. \end{cases} \tag{8}$$

where $\rho(t,x)$ is defined in Assumption 2.2.

With regard to the stability of the uncertain dynamical system, we may state the following proposition.

Proposition 3.1. Given the system (3), if Assumptions 2.1 and 2.2 are valid, the condition $\sigma^T\dot{\sigma} < 0$ is satisfied by employing the control law (5).

Proof. Refer to Walcott *et al.* (1989) as it is similar.

In the design of this class of feedback controller, a continuous positive scalar valued function $\rho(\cdot,\cdot)$ satisfying Assumption 2.2 is an important key used to guarantee the stability of uncertain dynamical systems. But, sometimes it may not be easily obtained due to the complexity of the structure of the uncertainty. Especially, the magnitude of the extraneous disturbance cannot be simply estimated.

Our goal is to introduce an adaptive scheme which is capable of performing an estimation of the upper bound of the norm $\|e(t,x)\|$ and to design a variable structure controller using this adaptive upper bound. For these purposes, we state the modification of Assumption 2.2.

Assumption 3.1. There are positive constants (Yoo and Chung, 1992), c_0 and c_1 such that

$$\|e(t,x)\| \le c_0 + c_1\|x\| = \rho(t,x) \quad \text{for all } (t,x) \in \mathbb{R} \times \mathbb{R}^n. \tag{9}$$

Now, u_N in the SMC control law (5) becomes

$$u_N = \begin{cases} -\dfrac{B^TS^T\sigma}{\|B^TS^T\sigma\|}\bar{\rho}(t,x), & \text{if } \sigma \neq 0, \\ 0 & \text{if } \sigma = 0, \end{cases} \tag{10}$$

therein, $\bar{\rho}(t,x)$ is the adaptive upper bound of the norm $\|e(t,x)\|$ and is synthesized by

$$\bar{\rho}(t,x) = \bar{c}_0(t,x) + \bar{c}_1(t,x)\|x\|, \tag{11}$$

where $\bar{c}_0(t,x)$ and $\bar{c}_1(t,x)$ are the estimated parameters about c_0 and c_1, respectively. An adaptation law for the upper bound of the norm $\|e(t,x)\|$ is defined as

$$\dot{\bar{c}}_0(t,x) \triangleq q_0\|B^TS^T\sigma\|, \tag{12}$$

$$\dot{\bar{c}}_1(t,x) \triangleq q_1\|B^TS^T\sigma\|\,\|x\|, \tag{13}$$

where q_0 and q_1 are adaptation gains with positive values. By choosing appropriate q_0 and q_1, the rate of parameter adaption can be adjusted. In theory, as the adaptation gains are getting larger, the rate of parameter adaptation is getting higher. In practice, the adaptation gains are limited by the bound of control input and other practical considerations.

Then, we may state the following proposition.

Proposition 3.2. Given system (1), if Assumptions 2.1 and 3.1 are valid, $\sigma(x) = 0$ is asymptotically stable by employing the control (5) with u_N given in equation (10) and the adaptation laws (12) and (13).

Proof. See Yoo and Chung (1992).

Remark. By Proposition 3.2, once the system state reaches the switching surface, it slides along the switching surface and the system response depends thereafter only on the gradient of the switching surface. Thus, if the switching surface is chosen such that the system restricted to the switching surface is asymptotically stable, then the proposed control (5) with u_N in equation (10) renders the system (1) asymptotically stable.

A drawback to the control law given in equation (10) is that it is discontinuous about the switching surface $\sigma(x) = 0$. This characteristic induces an undesirable chattering problem. For practical implementations the controller must be smoothed. One way to smooth the control is to introduce a boundary layer about the switching surface, as suggested in Yoo and Chung (1992):

$$u_N = \begin{cases} -\dfrac{B^TS^T\sigma}{\|B^TS^T\sigma\|}\bar{\rho}(t,x) & \text{if } \|B^TS^T\sigma\| > \varepsilon, \\ -\dfrac{B^TS^T\sigma}{\varepsilon}\bar{\rho}(t,x) & \text{if } \|B^TS^T\sigma\| \le \varepsilon, \end{cases} \tag{14}$$

$$\bar{\rho}(t,x) = \bar{c}_0(t,x) + \bar{c}_1(t,x)\|x\|, \tag{15}$$

$$\dot{\bar{c}}_0(t,x) = q_0\|B^TS^T\sigma\|, \tag{16}$$

$$\dot{\bar{c}}_1(t,x) = q_1\|B^TS^T\sigma\|\|x\|, \tag{17}$$

where ε is a small positive value. As ε approaches zero, the performance of this boundary layer control law can be made arbitrarily close to that of original control law. However, we should point out that in this case the estimated gains $\bar{c}_0(t,x)$ and $\bar{c}_1(t,x)$ may become unbounded in the boundary layer since the restriction to the sliding surface can not always be achieved. This argument has also been verified by the simulation results.

In order to address this problem, we propose a new smoothed SMC control law taking account of the boundary layer effect, i.e. u_N in equation (5) is modified as

$$u_N = \begin{cases} -\dfrac{B^TS^T\sigma}{\|B^TS^T\sigma\|}\bar{\rho}(t,x) & \text{if } \bar{\rho}\|B^TS^T\sigma\| > \varepsilon, \\ -\dfrac{B^TS^T\sigma}{\varepsilon}\bar{\rho}^2(t,x) & \text{if } \bar{\rho}\|B^TS^T\sigma\| \leqslant \varepsilon, \end{cases} \tag{18}$$

where

$$\bar{\rho}(t,x) = \bar{c}_0(t,x) + \bar{c}_1(t,x)\|x\|, \tag{19}$$

$$\dot{\bar{c}}_0(t,x) = q_0(-\psi_0\bar{c}_0 + \|B^TS^T\sigma\|), \tag{20}$$

$$\dot{\bar{c}}_1(t,x) = q_1(-\psi_1\bar{c}_1 + \|B^TS^T\sigma\|\|x\|), \tag{21}$$

where ε is a small positive value, ψ_0 and ψ_1 are constants chosen by the designer. The performance of this boundary layer control law can be stated by the following proposition:

Proposition 3.3. Given system (1), if Assumptions 2.1 and 3.1 are valid, then the control law (5) with u_N in equation (18) is continuous and in the closed-loop system $\sigma(x)$ and all signals are uniformly ultimately bounded.

Proof. Consider the following Lyapunov function:

$$2V = \sigma^T\sigma + q_0^{-1}\tilde{c}_0^2 + q_1^{-1}\tilde{c}_1^2, \tag{22}$$

where $\tilde{c}_0(t,x) = \bar{c}_0(t,x) - c_0$ and $\tilde{c}_1(t,x) = \bar{c}_1(t,x) - c_1$ are parameter adaptation errors. Differentiating V with respect to time yields

$$\dot{V} = \sigma^T\dot{\sigma} + q_0^{-1}\tilde{c}_0\dot{\bar{c}}_0 + q_1^{-1}\tilde{c}_1\dot{\bar{c}}_1. \tag{23}$$

If $\|B^TS^T\sigma\| > \varepsilon/\bar{\rho}$, then equation (23) can be written as

$$\begin{aligned}
\dot{V} &= \sigma^T[-K\sigma + SB(u_N + e)] + q_0^{-1}\tilde{c}_0\dot{\bar{c}}_0 + q_1^{-1}\tilde{c}_1\dot{\bar{c}}_1 \\
&= -\sigma^TK\sigma - \|B^TS^T\sigma\|(\bar{c}_0 + \bar{c}_1\|x\|) + \sigma^TSBe \\
&\quad + q_0^{-1}\tilde{c}_0\dot{\bar{c}}_0 + q_1^{-1}\tilde{c}_1\dot{\bar{c}}_1 \\
&\le -\sigma^TK\sigma - \|B^TS^T\sigma\|(\bar{c}_0 + \bar{c}_1\|x\|) + \|B^TS^T\sigma\|\|e\| \\
&\quad + q_0^{-1}\tilde{c}_0\dot{\bar{c}}_0 + q_1^{-1}\tilde{c}_1\dot{\bar{c}}_1 \\
&\le -\sigma^TK\sigma + \tilde{c}_0(q_0^{-1}\dot{\bar{c}}_0 - \|B^TS^T\sigma\|) \\
&\quad + \tilde{c}_1(q_1^{-1}\dot{\bar{c}}_1 - \|B^TS^T\sigma\|\,\|x\|) \\
&= -\sigma^TK\sigma - \psi_0\tilde{c}_0\bar{c}_0 - \psi_1\tilde{c}_1\bar{c}_1 \\
&= -\sigma^TK\sigma - \psi_0(\tfrac{1}{2}c_0 - \bar{c}_0)^2 - \psi_1(\tfrac{1}{2}c_1 - \bar{c}_1)^2 \\
&\quad + \tfrac{1}{4}\psi_0c_0^2 + \tfrac{1}{4}\psi_1c_1^2 \\
&\le -\sigma^TK\sigma - \psi_0(\tfrac{1}{2}c_0 - \bar{c}_0)^2 - \psi_1(\tfrac{1}{2}c_1 - \bar{c}_1)^2 + \varepsilon_1,
\end{aligned} \tag{24}$$

where $\varepsilon_1 \triangleq \frac{1}{4}\psi_0c_0^2 + \frac{1}{4}\psi_1c_1^2$.

If $\|B^TS^T\sigma\| \le \varepsilon/\bar{\rho}$, then equation (23) can be written as

$$\begin{aligned}
\dot{V} &= \sigma^T[-K\sigma + SB(u_N + e)] + q_0^{-1}\tilde{c}_0\dot{\bar{c}}_0 + q_1^{-1}\tilde{c}_1\dot{\bar{c}}_1 \\
&\le -\sigma^TK\sigma - \frac{\|B^TS^T\sigma\|^2}{\varepsilon}\bar{\rho}^2 + \|B^TS^T\sigma\|\,\|e\| \\
&\quad + q_0^{-1}\tilde{c}_0\dot{\bar{c}}_0 + q_1^{-1}\tilde{c}_1\dot{\bar{c}}_1 \\
&\le -\sigma^TK\sigma - \frac{\|B^TS^T\sigma\|^2}{\varepsilon}\bar{\rho}^2 + \|B^TS^T\sigma\|\rho + q_0^{-1}\tilde{c}_0\dot{\bar{c}}_0 \\
&\quad + q_1^{-1}\tilde{c}_1\dot{\bar{c}}_1 \\
&= -\sigma^TK\sigma - \frac{\|B^TS^T\sigma\|^2}{\varepsilon}\bar{\rho}^2 + \|B^TS^T\sigma\|\bar{\rho} - \psi_0\tilde{c}_0\bar{c}_0 \\
&\quad - \psi_1\tilde{c}_1\bar{c}_1.
\end{aligned} \tag{25}$$

When $\|B^TS^T\sigma\|\bar{\rho} = \varepsilon/2$, the term $[-\|B^TS^T\sigma\|^2/\varepsilon]\bar{\rho}^2 + \|B^TS^T\sigma\|\bar{\rho}]$ reaches the maximum value of $\varepsilon/2$. Thus we may write as follows:

$$\begin{aligned}
\dot{V} &\le -\sigma^TK\sigma + \tfrac{1}{2}\varepsilon - \psi_0\tilde{c}_0\bar{c}_0 - \psi_1\tilde{c}_1\bar{c}_1 \\
&= -\sigma^TK\sigma - \psi_0(\tfrac{1}{2}c_0 - \bar{c}_0)^2 - \psi_1(\tfrac{1}{2}c_1 - \bar{c}_1)^2 + \frac{\varepsilon}{2} + \varepsilon_1 \\
&\le -\sigma^TK\sigma - \psi_0(\tfrac{1}{2}c_0 - \bar{c}_0)^2 - \psi_1(\tfrac{1}{2}c_1 - \bar{c}_1)^2 + \varepsilon_2
\end{aligned} \tag{26}$$

where $\varepsilon_2 \triangleq \frac{1}{2}\varepsilon + \varepsilon_1$.

Based on equations (24) and (26), the uniform ultimate boundedness thus follows using the result and terminology in Corless and Leitmann (1981).

Remarks. (1) Unlike u_N in equation (14), u_N in equation (18) uses a new criterion $\|B^TS^T\sigma\| > \varepsilon/\bar{\rho}$ (or $\|B^TS^T\sigma\| \le \varepsilon/\bar{\rho}$) instead of $\|B^TS^T\sigma\| > \varepsilon$ (or $\|B^TS^T\sigma\| \le \varepsilon$). Thus, the boundedness of the system can be guaranteed.

(2) The adaptive laws (20) and (21) are similar to the so-called σ-modification in Ioannou and Kokotovic (1984), though different purposes are pursued. Thus, we have developed an implementable controller which can be described as a combination of SMC and σ-modification adaptive control. As a matter of fact, we can also choose ψ_0 and ψ_1 as switching-σ functions Ioannou and Tsakalis (1986)

$$\psi_0 = \begin{cases} 0, & \bar{c}_0| \le c_0^0, \\ \psi_0^0(\frac{|\bar{c}_0|}{c_0^0} - 1), & c_0^0 < |\bar{c}_0| \le 2c_0^0, \\ \psi_0^0, & 2c_0^0 < |\bar{c}_0|, \end{cases} \tag{27}$$

$$\psi_1 = \begin{cases} 0, & |\bar{c}_1| \le c_1^0, \\ \psi_1^0(\frac{|\bar{c}_0|}{c_1^0} - 1), & 0_1 < |\bar{c}_1| \le 2c_1^0, \\ \psi_1^0, & 2c_1^0 < |\bar{c}_1|, \end{cases} \tag{28}$$

where ψ_0^0 and ψ_1^0 are positive constants; c_0^0 and c_1^0 satisfy $c_0^0 > c_0$ and $c_1^0 > c_1$. In this case, we can still prove the boundedness of the closed loop system. The benefit of using switching-σ is that less error bounds may be expected. It should be mentioned that even with very conservative choices of c_0^0 and c_1^0, they are only related to the transition of ψ_0 and ψ_1, and not to the control gains. This differs from the case where the upper bounds are required to be known. We should note that our goal in this paper is to develop an implementable adaptive SMC strategy in a simpler setting that reveals its essential feature. This is the motivation for simply using ψ_0 and ψ_1 as constants in the theorem.

4. *Simulation results*

In order to illustrate the proposed controller, we consider the position control of the pendulum (Yoo and Chung, 1992) shown in Fig. 1, which has the variable length $l(\theta)$. Substituting suitable parameter values, the equation of motion can be written as

$$\begin{aligned}
\ddot{\theta} &= 0.5\sin\theta(1 + 0.5\cos\theta)\dot{\theta}^2\delta(\theta) - 10\sin\theta(1 + \cos\theta)\delta(\theta) \\
&\quad + T\delta(\theta) + v(t)\cos\theta,
\end{aligned} \tag{29}$$

where $\delta(\theta) = 0.25(\cos\theta + 2)^2$ and the disturbance $v(t) = 2\cos(3t)$.

Assuming $x^T = [x_1, x_2] = [\theta, \dot{\theta}]$ and $u = T$, then equaiton (29) can be written as follows:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u + e(t,x)). \tag{30}$$

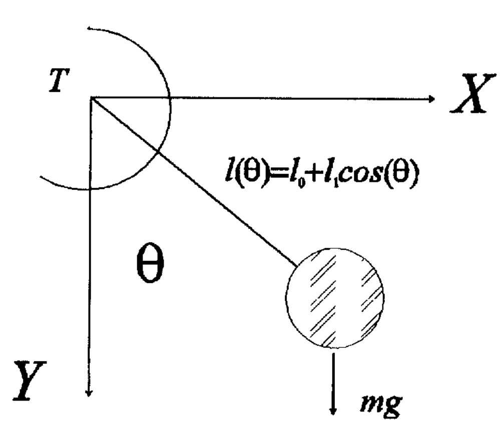

Fig. 1. System model: $l_1/l_0 = 0.5$, $g/l_0 = 10$, and $ml_0^2 = 1$.

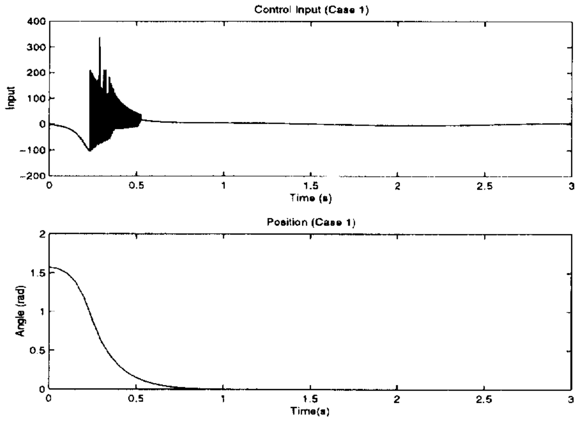

Fig. 2. Case 1: $\varepsilon = 0.5$, $q_0 = 5$, and $q_1 = 5$.

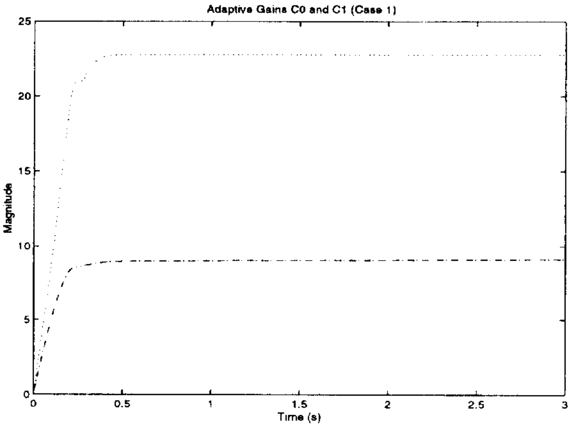

Fig. 3. $\bar{c}_0$, and $\bar{c}_1$ of case 1: $\varepsilon = 0.5$, $q_0 = 5$, and $q_1 = 5$.

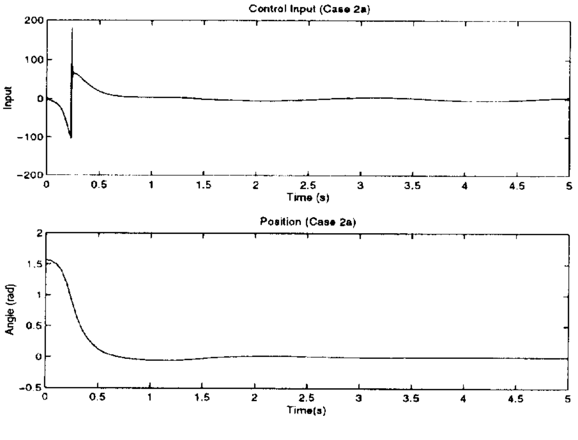

Fig. 4. Case 2a: $\varepsilon = 60e^{-1.5t} + 0.2$, $q_0 = q_1 = 5$, $\psi_0 = 0.01$ and $\psi_1 = 0.005$.

where

$$e(t, x) = [\sin x_1(-10(1 + \cos x_1) + (1 - \delta(x_1))\, u + 0.5(1 + 0.5\cos x_1)\cdot x_2^2)]/[\delta(x_1)] + v(t)\cos x_1. \quad (31)$$

Following [10] we choose the switching surface $\sigma(x) = 7x_1 + x_2 = 0$. Then, from equation (5), the VSC is determined as $u = -K\sigma(x) - 7x_2 + u_N$.

Now, for $x(0) = [\pi/2, 0]T$, let us set $[\bar{c}_0, \bar{c}_1] = [0, 0]$, $K = 0.1$, $[q_0, q_1] = [5, 5]$, and observe the system response.

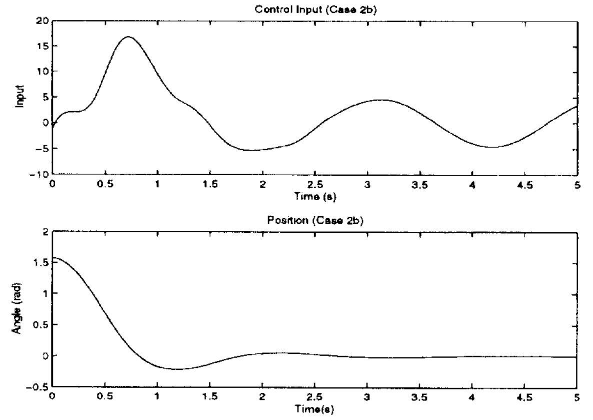

Fig. 5. Case 2b: $\varepsilon = 60e^{-1.5t} + 0.2$, and $q_0 = q_1 = 5$.

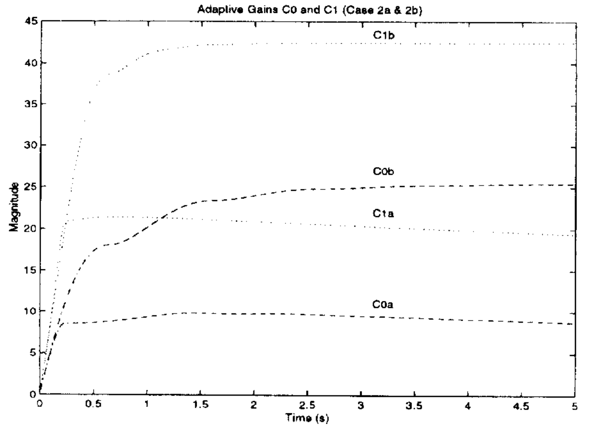

Fig. 6. $\bar{c}_0$, and $\bar{c}_1$ of case 2a and b.

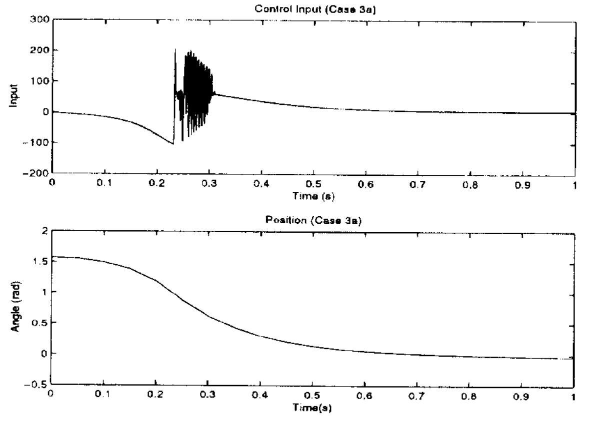

Fig. 7. Case 3a: $\varepsilon = 20e^{-t} + 0.02$, $q_0 = q_1 = 5$, $\psi_0 = 0.01$, and $\psi_1 = 0.005$.

Case 1. Using the controller (5) with u_N in equation (14) where $\varepsilon = 0.5$, the results are shown in Figs. 2 and 3. From these results, we see that $\bar{c}_0$ and $\bar{c}_1$ tend to grow unboundedly as shown in the analysis, and also there is initial control chatter.

Case 2: Now the new controller (5) with u_N in equation (18) is used to see the performance of the system, where $[\psi_0, \psi_1]$ are chosen as $[\psi_0, \psi_1] = [0.01, 0.005]$. To remove the chattering, in this simulation we choose a time-varying boundary layer, i.e. $\varepsilon = 60e^{-1.5t} + 0.2$. For comparison, the controller (5) with u_N in equation (14) is also used with exactly the same parameters as

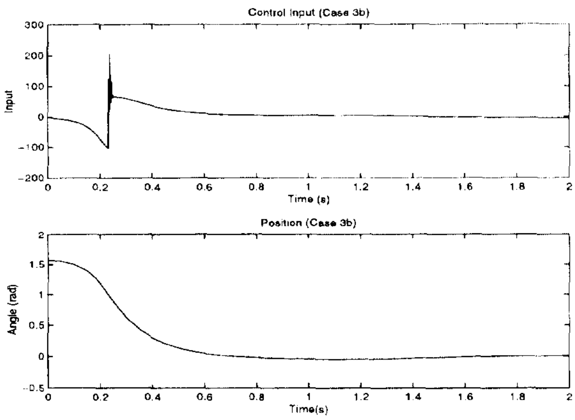

Fig. 8. Case 3b: $\varepsilon = 50e^{-1.5t} + 0.02$, $q_0 = q_1 = 5$, $\psi_0 = 0.01$, and $\psi_1 = 0.005$.

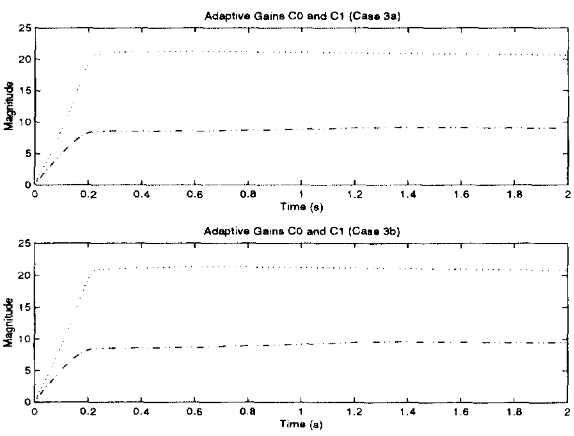

Fig. 9. $\bar{c}_0$, and $\bar{c}_1$ of case 3a and b.

in equation (18). Results are shown in Figs. 4–6, where a stands for results with u_N in equation (18) and b for u_N in equation (14). From Fig. 6 we see that $\bar{c}_0(b)$ and $\bar{c}_1(b)$ tend to grow unbounded while $\bar{c}_0(a)$ and $\bar{c}_1(a)$ are bounded and thus verify the proposed method.

Case 3: Here we can see that the choice of ε influences the performance of the system. Figures 7–9 show the system responses for different choices of ε while the other controller parameters are kept the same. Therein, a stands for the case $\varepsilon = 20e^{-t} + 0.02$ and b for $\varepsilon = 50e^{-1.5t} + 0.02$. From these results we see that we need to choose a suitable ε and that the choice of an optimal ε is worthy of further research.

5. *Conclusions*

For SMC of uncertain dynamical systems, the bound on the uncertainty is an important parameter and may not be easily obtained due to several causes. Therefore, adaptive methods were introduced to estimate such bounds. For practical implementations the controller must be smoothed within a boundary layer. In this case the estimated gain may become unbounded in the boundary layer since the restriction to the sliding surface cannot always be achieved. Focusing on this problem, an improved adaptation law is proposed for the upper bound on the uncertainty and a new controller is designed which can guarantee the boundedness of the closed-loop system and can be described as a combination of SMC law and σ-modification adaptive law. From the simulation results, we showed that the proposed method removes the chatter (Chen and Mita, 1993) and controls the uncertain dynamical systems using boundary layer control.

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