Square and add these two equations to get:

$$L_2^2 + L_3^2 + 2L_2L_3c_3 = \alpha^2 + p_z^2$$

$$c_3 = \frac{1}{2L_2L_3}(\alpha^2 + p_z^2 - L_2^2 - L_3^2)$$

$$s_3 = \pm\sqrt{1-c_3^2}$$

$$\Rightarrow \qquad \theta_3 = a\tan 2(s_3, c_3)$$

Finally,

$$L_3c_{23} = \alpha - L_2c_2$$

$$L_3s_{23} = p_z - L_2s_2$$

$$\Rightarrow \qquad \theta_2 = a\tan 2(p_z - L_2s_2, \alpha - L_2c_2) - \theta_3$$

Solutions #4

5.2) FROM EXERCISE 3.3 WE HAVE:

$$ {}^0_3T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & L_1c_1 + L_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & L_1s_1 + L_2s_1c_2 \\ s_{23} & c_{23} & 0 & L_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

AND:

$$ {}^3_4T = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^0_4T = {}^0_3T \; {}^3_4T $$

WE COULD THEN FIND ${}^0J(\underline{\theta})$ QUITE EASILY BY DIFFERENTIATING ${}^0P_{4ORG}$. FINALLY, ${}^4J(\underline{\theta})$ CAN BE CALCULATED AS ${}^4_0R\,{}^0J(\underline{\theta})$. THIS MIGHT BE TEDIOUS, SO LETS TRY "STANDARD" VELOCITY PROPAGATION AS DONE IN THE TEXT:

$$ {}^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \qquad {}^1V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^2\omega_2 = {}^2_1R\,{}^1\omega_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} $$

$$ {}^2\omega_2 = \begin{bmatrix} s_2\dot{\theta}_1 \\ c_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad {}^2V_2 = {}^2_1R\left({}^1V_1 + {}^1\omega_1 \times {}^1P_2\right) $$

$$ {}^2V_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ L_1\dot{\theta}_1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ -L_1\dot{\theta}_1 \end{bmatrix} $$

$$ {}^3\omega_3 = {}^3_2R\,{}^2\omega_2 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_2\dot{\theta}_1 \\ c_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} $$

$$ {}^3\omega_3 = \begin{bmatrix} s_{23}\dot{\theta}_1 \\ c_{23}\dot{\theta}_1 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} \qquad {}^3V_3 = {}^3_2R\left({}^2V_2 + {}^2\omega_2 \times {}^2P_3\right) $$

$$ {}^3V_3 = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ -L_1\dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_2\dot{\theta}_2 \\ -L_2c_2\dot{\theta}_1 \end{bmatrix} \right) $$

$$ {}^3V_3 = \begin{bmatrix} s_3 L_2 \dot\theta_2 \\ c_3 L_2 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 c_2 \dot\theta_1 \end{bmatrix} \qquad {}^4\omega_4 = {}^3\omega_3 $$

$$ {}^4V_4 = {}^4_3R\left({}^3V_3 + {}^3\omega_3 \times {}^3P_4\right) = {}^3V_3 + {}^3\omega_3 \times {}^3P_4 $$

$$ = \begin{bmatrix} s_3 L_2 \dot\theta_2 \\ c_3 L_2 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 c_2 \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_3(\dot\theta_2+\dot\theta_3) \\ -L_3 c_{23}\dot\theta_1 \end{bmatrix} = \begin{bmatrix} s_3 L_2 \dot\theta_2 \\ c_3 L_2 \dot\theta_2 + L_3(\dot\theta_2+\dot\theta_3) \\ -L_1\dot\theta_1 - L_2 c_2\dot\theta_1 - L_3 c_{23}\dot\theta_1 \end{bmatrix} $$

$$ \therefore \quad {}^4J(\underline{\theta}) = \begin{bmatrix} 0 & s_3 L_2 & 0 \\ 0 & c_3 L_2 + L_3 & L_3 \\ (-L_1 - L_2 c_2 - L_3 c_{23}) & 0 & 0 \end{bmatrix} $$

5.3) FIRST, VELOCITY ANALYSIS: (THIS PART IS SAME AS EXER. 5.2)

$$ {}^1\omega_1 = {}^1_0R\, {}^0\omega_0 + \dot\theta_1\, {}^1\hat{Z}_1 = \dot\theta_1 \hat{Z} $$

$$ {}^1V_1 = {}^1_0R\left({}^0V_0 + {}^0\omega_0 \times {}^0P_1\right) = 0 $$

$$ {}^2\omega_2 = {}^2_1R\, {}^1\omega_1 + \dot\theta_2\, {}^2\hat{Z}_2 $$

$$ {}^2\omega_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} s_2\dot\theta_1 \\ c_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} $$

$$_2 = {}^2_1R\left({}^1v_1 + {}^1\omega_1 \times {}^1P_2\right)$$

$$_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} \times \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ -s_2 & 0 & c_2 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ L_1\dot\theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix}$$

$${}^3\omega_3 = {}^3_2R\,{}^2\omega_2 + \dot\theta_3\,{}^3\hat{z}_3$$

$${}^3\omega_3 = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} s_2\dot\theta_1 \\ c_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} s_2c_3\dot\theta_1 + c_2s_3\dot\theta_1 \\ -s_2s_3\dot\theta_1 + c_2c_3\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{bmatrix}$$

$${}^3\omega_3 = \begin{bmatrix} s_{23}\dot\theta_1 \\ c_{23}\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{bmatrix}$$

$${}^3v_3 = {}^3_2R\left({}^2v_2 + {}^2\omega_2 \times {}^2P_3\right)$$

$${}^3v_3 = {}^3_2R\left(\begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} + \begin{bmatrix} s_2\dot\theta_1 \\ c_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} \times \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}\right)$$

$${}^3v_3 = \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ L_2\dot\theta_2 \\ -L_1\dot\theta_1 - L_2c_2\dot\theta_1 \end{bmatrix} = \begin{bmatrix} L_2s_3\dot\theta_2 \\ L_2c_3\dot\theta_2 \\ -L_1\dot\theta_1 - L_2c_2\dot\theta_1 \end{bmatrix}$$

$${}^4\omega_4 = {}^4_3R\,{}^3\omega_3 + 0 \quad ; \quad {}^4_3R = I \quad ; \quad {}^4\omega_4 = {}^3\omega_3$$

$$V_4 = {}^4_3R\left({}^3V_3 + {}^3\omega_3 \times {}^3P_4\right)$$

$$V_4 = {}^4_3R\left(\begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 \end{bmatrix} + \begin{bmatrix} S_{23}\dot\theta_1 \\ C_{23}\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{bmatrix} \times \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix}\right)$$

$${}_4 = \left(\begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_3(\dot\theta_2 + \dot\theta_3) \\ -L_3 C_{23}\dot\theta_1 \end{bmatrix}\right)$$

$${}_4 = \begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 + L_3(\dot\theta_2 + \dot\theta_3) \\ -L_1\dot\theta_1 - L_2 C_2\dot\theta_1 - L_3 C_{23}\dot\theta_1 \end{bmatrix}$$

$$, = {}^4J(\underline{\theta})\,\underline{\dot\theta}$$

$${}^4J(\underline{\theta}) = \begin{bmatrix} 0 & L_2 S_3 & 0 \\ 0 & L_2 C_3 + L_3 & L_3 \\ -L_1 - L_2 C_2 - L_3 C_{23} & 0 & 0 \end{bmatrix}$$

T, USING FORCE ANALYSIS:

$$F_4 = \begin{bmatrix} f_x \\ f_Y \\ f_z \end{bmatrix} \qquad {}^4N_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$ {}^{3}F_3 = {}^{3}_{4}R\ {}^{4}F_4 = \begin{bmatrix} f_x \\ f_Y \\ f_z \end{bmatrix} $$

$$ {}^{3}N_3 = {}^{3}_{4}R\ {}^{4}N_4 + {}^{3}P_4 \times {}^{3}F_3 = \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} f_x \\ f_Y \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ -L_3 f_z \\ L_3 f_Y \end{bmatrix} $$

$$ {}^{2}F_2 = {}^{2}_{3}R\ {}^{3}F_3 = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_x \\ f_Y \\ f_z \end{bmatrix} = \begin{bmatrix} c_3 f_x - s_3 f_Y \\ s_3 f_x + c_3 f_Y \\ f_z \end{bmatrix} $$

$$ {}^{2}N_2 = {}^{2}_{3}R\ {}^{3}N_3 + {}^{2}P_3 \times {}^{2}F_2 $$

$$ {}^{2}N_2 = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l_3 f_z \\ l_3 f_Y \end{bmatrix} + \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} c_3 f_x - s_3 f_Y \\ s_3 f_x + c_3 f_Y \\ f_z \end{bmatrix} $$

$$ {}^{2}N_2 = \begin{bmatrix} L_3 s_3 f_z \\ -L_2 f_z - L_3 c_3 f_z \\ L_2 (s_3 f_x + c_3 f_Y) + L_3 f_Y \end{bmatrix} $$

$$ {}^{1}F_1 = {}^{1}_{2}R\ {}^{2}F_2 = \begin{bmatrix} c_2 & -s_2 & 0 \\ 0 & 0 & -1 \\ s_2 & c_2 & 0 \end{bmatrix} \begin{bmatrix} c_3 f_x - s_3 f_Y \\ s_3 f_x + c_3 f_Y \\ f_z \end{bmatrix} $$

$$ {}^{1}F_1 = \begin{bmatrix} c_2 (c_3 f_x - s_3 f_Y) - s_2 (s_3 f_x + c_3 f_Y) \\ -f_z \\ s_2 (c_3 f_x - s_3 f_Y) + c_2 (s_3 f_x + c_3 f_Y) \end{bmatrix} $$

$$ {}^{1}N_1 = {}^{1}_{2}R \, {}^{2}N_2 + {}^{1}P_2 \times {}^{1}F_1 $$

$$ {}^{1}N_1 = \begin{bmatrix} c_2 & -s_2 & 0 \\ 0 & 0 & -1 \\ s_2 & c_2 & 0 \end{bmatrix} \begin{bmatrix} l_3 s_3 f_z \\ -l_2 f_z - l_3 c_3 f_z \\ l_2 (s_3 f_x + c_3 f_Y) + l_3 f_Y \end{bmatrix} + \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \times {}^{1}F_1 $$

$$ {}^{1}N_1 = \begin{bmatrix} c_2 l_3 s_3 f_z + s_2 l_2 f_z + l_2 s_2 c_3 f_z \\ -l_2 (s_3 f_x + c_3 f_Y) - l_3 f_Y \\ l_3 s_2 s_3 f_z - l_2 c_2 f_z - l_3 c_2 c_3 f_z \end{bmatrix} $$

$$ + \begin{bmatrix} 0 \\ -l_1 s_2 (c_3 f_x - s_3 f_Y) - l_2 c_2 (s_3 f_x + c_3 f_Y) \\ -l_1 f_z \end{bmatrix} $$

TO COMPUTE TORQUES, TAKE THE Z-COMPONENT OF THE ${}^{i}N_i$:

$$ \tau_1 = \left[-l_1 - l_2 c_2 + l_3 (s_2 s_3 - c_2 c_3) \right] f_z $$

$$ \tau_2 = l_2 s_3 f_x + (l_2 c_3 + l_3) f_Y $$

$$ \tau_3 = l_3 f_Y $$

$$ \underline{\tau} = {}^{4}J^{T}(\underline{\theta}) \begin{bmatrix} f_x \\ f_Y \\ f_z \end{bmatrix} $$

WHICH LEADS TO SAME EXPRESSION AS BEFORE FOR ${}^{4}J(\underline{\theta})$.

FINALLY, BY DIFFERENTIATION OF KINEMATIC EQUATIONS:

$$
{}^{0}P_{4ORG} = \begin{bmatrix} L_1 C_1 + L_2 C_1 C_2 + L_3 C_1 C_{23} \\ L_1 S_1 + L_2 S_1 C_2 + L_3 S_1 C_{23} \\ L_2 S_2 + L_3 S_{23} \end{bmatrix} \triangleq P
$$

$$
{}^{0}J(\underline{\theta}) = \begin{bmatrix} \frac{\partial P_x}{\partial \theta_1} & \frac{\partial P_x}{\partial \theta_2} & \frac{\partial P_x}{\partial \theta_3} \\ \frac{\partial P_Y}{\partial \theta_1} & \frac{\partial P_Y}{\partial \theta_2} & \frac{\partial P_Y}{\partial \theta_3} \\ \frac{\partial P_z}{\partial \theta_1} & \frac{\partial P_z}{\partial \theta_2} & \frac{\partial P_z}{\partial \theta_3} \end{bmatrix}
$$

$$
{}^{0}J(\underline{\theta}) = \begin{bmatrix} -L_1 S_1 - L_2 S_1 C_2 - L_3 S_1 C_{23} & -L_2 C_1 S_2 - L_3 C_1 S_{23} & -L_3 C_1 S_{23} \\ L_1 C_1 + L_2 C_1 C_2 + L_3 C_1 C_{23} & L_2 S_1 S_2 - L_3 S_1 S_{23} & -L_3 S_1 S_{23} \\ 0 & L_2 C_2 + L_3 C_{23} & L_3 C_{23} \end{bmatrix}
$$

$$
{}^{0}V_4 = {}^{0}J(\underline{\theta})\,\dot{\underline{\theta}} \quad ; \quad {}^{4}V_4 = \underbrace{{}^{4}_{0}R \; {}^{0}J(\underline{\theta})}_{{}^{4}J(\underline{\theta})}\,\dot{\underline{\theta}}
$$

$$
{}^{4}_{0}R = \begin{bmatrix} C_1 C_{23} & S_1 C_{23} & S_{23} \\ -C_1 S_{23} & -S_1 S_{23} & C_{23} \\ S_1 & -C_1 & 0 \end{bmatrix}
$$

MULTIPLYING OUT ${}^4_0R\,{}^0J(\underline{\theta})$ IS TEDIOUS, BUT SURE ENOUGH, IT LEADS AGAIN TO THE SAME EXPRESSION FOR ${}^4J(\underline{\theta})$.

5.7) SEE FIGURE 11.9 – IT MUST BE A PURELY CARTESIAN MANIPULATOR:

$$\underline{v} = J\dot{\underline{\theta}} \quad ; \quad \underline{v} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\underline{\theta}}$$

MUST HAVE 3 ORTHOGONAL PRISMATIC JOINTS, SO $\dot{\underline{\theta}} = [\dot{d}_1 \;\; \dot{d}_2 \;\; \dot{d}_3]^T$.

5.8) THE JACOBIAN OF THIS 2-LINK IS:

$${}^3J(\underline{\theta}) = \begin{bmatrix} L_1S_2 & 0 \\ L_1C_2 + L_2 & L_2 \end{bmatrix}$$

AN ISOTROPIC POINT EXISTS IF ${}^3J = \begin{bmatrix} L_2 & 0 \\ 0 & L_2 \end{bmatrix}$

SO, $L_1S_2 = L_2$

$L_1C_2 + L_2 = 0$

^{2}, $S_2 = \frac{L_2}{L_1}$ $\quad C_2 = \frac{-L_2}{L_1}$

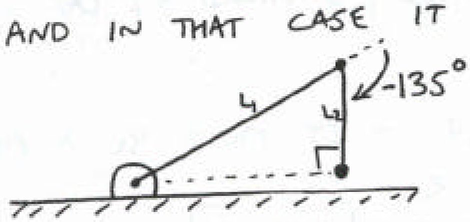
OW $S_2^2 + C_2^2 = 1$, SO $\left(\frac{L_2}{L_1}\right)^2 + \left(\frac{-L_2}{L_1}\right)^2 = 1$

R $L_1^2 = 2L_2^2 \rightarrow L_1 = \sqrt{2}\, L_2$

NDER THIS CONDITION $S_2 = \frac{1}{\sqrt{2}} = \pm .707$

ND $C_2 = -.707$

∴ AN ISOTROPIC POINT EXISTS IF $L_1 = \sqrt{2}\, L_2$
AND IN THAT CASE IT EXISTS WHEN $\theta_2 = \pm 135°$



IN THIS CONFIGURATION, THE MANIPULATOR LOOKS MOMENTARILY LIKE A CARTESIAN MANIPULATOR.

5.9) UNSOLVED. A SMALL PART OF THE ANSWER CAN BE FOUND IN REFERENCE [7] OF CHAP. 5.

5.10) $\underline{\tau} = {}^3J^T\, {}^3\underline{F} \quad \therefore\ {}^3\underline{F} = {}^3J^{-T}\, \underline{\tau}$

$${}^3J = \begin{bmatrix} L_1 S_2 & 0 \\ L_1 C_2 + L_2 & L_2 \end{bmatrix}$$

$${}^3J^T = \begin{bmatrix} L_1 S_2 & L_1 C_2 + L_2 \\ 0 & L_2 \end{bmatrix}$$

so,

$$ {}^{3}J^{-T} = \frac{1}{L_1 L_2 S_2} \begin{bmatrix} L_2 & -L_1 C_2 - L_2 \\ 0 & L_1 S_2 \end{bmatrix} $$

5.13) $\underline{\tau} = {}^{0}J^{T}(\underline{\theta})\, {}^{0}\underline{F}$

$$ \underline{\tau} = \begin{bmatrix} -L_1 S_1 - L_2 S_{12} & L_1 C_1 + L_2 C_{12} \\ -L_2 S_{12} & L_2 C_{12} \end{bmatrix} \begin{bmatrix} 10 \\ 0 \end{bmatrix} $$

$$ \tau_1 = -10 S_1 L_1 - 10 L_2 S_{12} $$

$$ \tau_2 = -10 L_2 S_{12} $$

5.15) THE KINEMATICS CAN BE DONE EASILY TO OBTAIN:

$$ {}^{0}P_{4ORG} = \begin{bmatrix} (d_2 + L_2 + L_3)\, s_1 \\ -(d_2 + L_2 + L_3)\, c_1 \\ 0 \end{bmatrix} $$

$$ {}^{0}\underline{v} = {}^{0}J\, \dot{\underline{\theta}} $$

$$ {}^{0}J = \begin{bmatrix} \frac{\partial\, {}^{0}P_{4ORGX}}{\partial \theta_1} & \frac{\partial\, {}^{0}P_{4ORGX}}{\partial \theta_2} & \frac{\partial\, {}^{0}P_{4ORGX}}{\partial \theta_3} \\ \frac{\partial\, {}^{0}P_{4ORGY}}{\partial \theta_1} & \frac{\partial\, {}^{0}P_{4ORGY}}{\partial \theta_2} & \frac{\partial\, {}^{0}P_{4ORGY}}{\partial \theta_3} \\ \frac{\partial\, {}^{0}P_{4ORGZ}}{\partial \theta_1} & \frac{\partial\, {}^{0}P_{4ORGZ}}{\partial \theta_2} & \frac{\partial\, {}^{0}P_{4ORGZ}}{\partial \theta_3} \end{bmatrix} $$

SO,

$$ {}^{0}J = \begin{bmatrix} (d_2 + L_2 + L_3)\, c_1 & s_1 & 0 \\ (d_2 + L_2 + L_3)\, s_1 & -c_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} $$