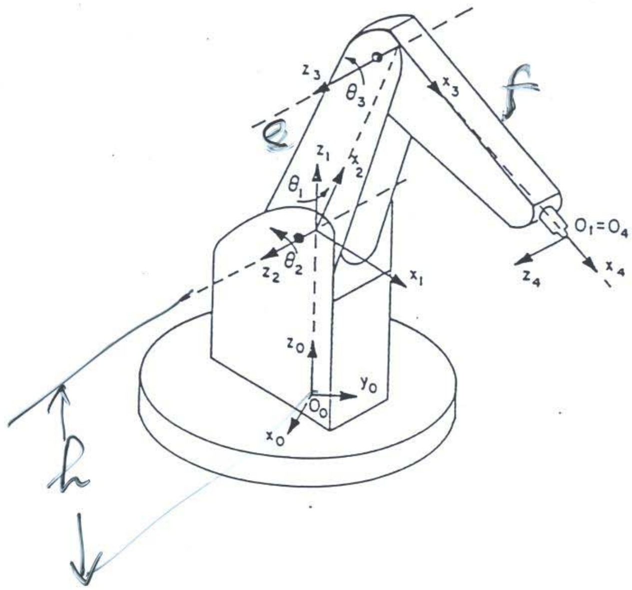

Figure 2: The Microbot

TABLE 1

Link Number i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0°	0	θ_1	h
2	90°	0	θ_2	0
3	0°	e	θ_3	0
4	0°	f	0°	0

$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & \vdots & 0 \\ s_1 & c_1 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & h \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}
\qquad
{}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & \vdots & 0 \\ 0 & 0 & -1 & \vdots & 0 \\ s_2 & c_2 & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & \vdots & e \\ s_3 & c_3 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}
\qquad
{}^3_4T = \begin{bmatrix} & & & \vdots & f \\ & I & & \vdots & 0 \\ & & & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}
$$

s_1

${}^2_3T$

$$
{}^0_4T = {}^0_1T.{}^1_2T.{}^3_4T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & \vdots & ec_1c_2 + fc_1c_{23} \\ s_1c_{23} & -s_1s_{23} & -c_1 & \vdots & es_1c_2 + fs_1c_{23} \\ s_{23} & c_{23} & 0 & \vdots & h + es_2 + fs_{23} \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}
$$

$-c_1$

In the inverse kinematics , specifications (p_x, p_y, p_z) in

$$
\boxed{{}^0_4T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \vdots & p_x \\ r_{21} & r_{22} & r_{23} & \vdots & p_y \\ r_{31} & r_{32} & r_{33} & \vdots & p_z \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix} \text{ are given}} \Longleftarrow \text{given}
$$

Find $\theta_1, \theta_2, \theta_3 = ?$

Solution:

Note that the robot has only 3 degree of freedom thus only gripper's position $[p_x \;\; p_y \;\; p_z]^T$ can be specified

$$p_x = ec_1c_2 + fc_1c_{23} \qquad (1)$$
$$p_y = es_1c_2 + fs_1c_{23} \qquad (2)$$
$$p_z = h + es_2 + fs_{23} \qquad (3)$$

(1) & (2) $\Rightarrow \quad s_1p_x = c_1p_y$

$\Rightarrow \quad \dfrac{s_1}{c_1} = \dfrac{p_y}{p_x}$

$\Rightarrow \quad \theta_1 = \operatorname{a\,tan2}(p_y.p_x)$

Another solution : $\hat{\theta}_1 = \theta_1 + 180^\circ$

To find θ_3, note

$p_z - h = es_2 + fs_{23} \qquad (4)$

$(1)^2+(2)^2+(4)^2 \Rightarrow$

$$p_x^{\,2} + p_y^{\,2} + (p_z - h)^2 = e^2c_1^{\,2}c_2^{\,2} + f^2c_1^{\,2}c_{23}^{\,2} + 2efc_1^{\,2}c_2c_{23} + e^2s_1^{\,2}c_2^{\,2} + f^2s_1^{\,2}c_{23}^{\,2} + 2efs_1^{\,2}c_2c_{23} + e^2s_2^{\,2} + f^2s_{23}^{\,2} + 2efs_2s_{23}$$
$$= e^2 + f^2 + 2ef(c_2c_{23} + s_2s_{23})$$
$$= e^2 + f^2 + 2efc_3$$

$$c_3 = \frac{p_x^{\,2} + p_y^{\,2} + (p_z - h)^2 - e^2 - f^2}{2ef} = k$$

$\Rightarrow \theta_3 = \operatorname{a\,tan2}(\pm\sqrt{1-k^2}, k)$

(3) $\Rightarrow \quad p_z - h = es_2 + fs_2c_3 + fc_2s_3$

$= (e + fc_3)s_2 + fs_3c_2$

$= k_1s_2 + k_2c_2$

where $k_1 = e + fc_3, k_2 = fs_3$ --------- (known)

then

$$\theta_2 = \operatorname{a\,tan2}(k_1, k_2) \pm \operatorname{a\,tan2}\left(\sqrt{k_1^{\,2} + k_2^{\,2} - (p_z - h)^2}, p_z - h\right)$$